

Game theory and market power

Josh Taylor

Based on section 6.1.3, 6.3 in *Convex Optimization of Power Systems*.

1 Market weaknesses

Recall

- Optimal power flow:

$$\begin{aligned} & \underset{p, \theta}{\text{minimize}} && \sum_i f_i(p_i) \\ & \text{subject to} && \\ & \lambda_i : && p_i = \sum_j b_{ij}(\theta_i - \theta_j) \\ & \chi_{ij} \geq 0 : && b_{ij}(\theta_i - \theta_j) \leq \bar{s}_{ij} \\ & && \underline{p}_i \leq p_i \leq \bar{p}_i \end{aligned}$$

- Prices: λ_i : the price at node i . Agent i solves:

$$\underset{p_i}{\text{minimize}} \quad f_i(p_i) - \lambda_i p_i$$

When does nodal pricing / microeconomics fail?

- Nonconvexity - the power flow equations, unit commitment.
- Bounded rationality - agent i has limited time, information, computing power - can't find optimal p_i .
- **Price-taker** assumption: agent i oblivious to their influence on λ_i .

2 Real-world examples

2.1 Enron Scandal, late 1990's to 2001

Overview:

- Energy trading, building power plants, natural gas
- Posterchild for electricity/energy markets
- Very shady accounting practices - see Wikipedia
- Gov. Gray Davis' ruined political career

California Electricity Crisis:

- Making power seem to be from out of state (where does your power come from?)
- Blocking transmission lines (over scheduling) to raise nodal prices
- Overall bad planning/market design
- Rolling blackouts in 2000, 2001, prices increase by factor of 20.

2.2 JPMorgan, 2010-2012

- Manipulative bidding strategies ...
- JPMorgan pays \$410 million in FERC settlement, 2013

2.3 Why?

- Lot's of markets have problems (healthcare, computer OS, diamonds)
- Failures of price-taker assumption in power
- Should we have power markets? Probably, but cautiously ...
- Were Enron, JPMorgan too smart?

Strategy:

- Markets need physically rigorous design
- Game theory helps identify vulnerabilities mathematically.

3 Game theory

Regular optimization:

$$\min_{x \in X} f(x).$$

Game theory:

$$\min_x f(x, y), \quad \min_y g(x, y)$$

Two players, know all about each other.

3.1 Example: prisoner's dilemma

Setup

- Two players, caught criminals
- Two actions: silence, or betray partner
- Made simultaneously (like rock paper scissors)

Payoffs:

- Both silent: both serve 1 year
- Both betray: both serve 3 years
- 1 silent, 1 betrays: silent 4 years, betrayer 0 years.

Anticipatory decisions:

- Both silent ... either improves by betraying
- 1 silent, 1 betrays ... silent improves by betraying
- Both betray ... no improvement for either

Nash equilibrium:

- Both players betray
- Stable under unilateral actions
- Worse than both silent

Strategic form games

- Players, $i = 1, \dots, n$
- Pure player strategies, \mathcal{S}_i .
- Player utility function $u_i(s)$, $s \in \mathcal{S} = \times_i \mathcal{S}_i$
- *Ordinary optimization with just one player*

s is a Pure Nash Eq. (PNE) if

$$u_i(s) \leq u_i(t, s_{-i}) \quad \text{for all } t \in \mathcal{S}_i.$$

PNE exists if

- $u_i(s)$ convex in s_i , continuous in s_{-i}
- \mathcal{S}_i convex and compact

Mixed strategy:

- \mathcal{M}_i set of PDFs over \mathcal{S}_i
- Mixed strategy $m \in \times_i \mathcal{M}_i$
- Expected payoff over m :

$$u_i(m) = \int_{\mathcal{S}} u_i(s) m(s) ds.$$

- MNE if

$$u_i(m) \leq u_i(t, m_{-i}) \quad \text{for all } t \in \mathcal{M}_i \quad (\mathcal{S}_i \text{ works})$$

Discussion

- PNE often don't exist.
- Uniqueness uncommon when it does exist.
- MNE describe real situations like sales.
- MNE almost always exist.
- Game theory PPAD complete - easier than NP-complete, still bad.

3.2 Bertrand competition

- Demand: d
- Prices: λ_i
- All demand goes to lowest price.

Equilibrium:

- If $\lambda_1 = \lambda_2 > 0$, $\lambda_1 - \epsilon$ is profitable for λ_1 .
- If $\lambda_1 > \lambda_2$, $\lambda_1 = \lambda_2 - \epsilon$ is profitable for λ_1 .
- Nash Eq: $\lambda_1 = \lambda_2 = 0$ (silly)

4 Load shifting with storage

- Time-varying, inelastic load $\delta(t)$, $t = 1, \dots, T$
- Generation cost $f(p) = \frac{a}{2}p^2 + bp$
- Market clearing price:

$$\begin{aligned}\lambda &= \frac{df(p)}{dp} \\ &= ap + b\end{aligned}$$

- N storages inject/extract $s_i(t)$ - arbitrage
- Net load: $\delta(t) - \sum_{i=1}^N s_i(t)$

Centralized problem,

$$\begin{aligned}\text{minimize}_s \quad & \sum_{t=1}^T f\left(\delta(t) - \sum_{i=1}^N s_i(t)\right) \\ \text{subject to} \quad & \sum_{t=1}^T s_i(t) = 0\end{aligned}$$

Optimal solutions:

$$\begin{aligned}s_i^c(t) &= \gamma_i (\delta(t) - \bar{\delta}) \\ \sum_{i=1}^N \gamma_i &= 1\end{aligned}$$

where the average demand is

$$\bar{\delta} = \frac{1}{T} \sum_{t=1}^T \delta(t).$$

- Net load curve: $\bar{\delta}$
- γ which storage allocation

Remove price-taker assumption. Market price:

$$\lambda(t) = a \left(\delta(t) - \sum_{i=1}^N s_i(t) \right) + b.$$

Storage payoffs:

$$\begin{aligned} & \underset{s_i}{\text{maximize}} && \sum_{t=1}^T \left(a \left(\delta(t) - \sum_{i=1}^N s_i(t) \right) + b \right) s_i(t) \\ & \text{subject to} && \sum_{t=1}^T s_i(t) = 0 \end{aligned}$$

- Coupling ... N -player game
- Quantity competition - Cournot
- PNE:

$$s_i^g(t) = \frac{1}{N+1} (\delta(t) - \bar{\delta})$$

- Flatter, but less so than centralized.
- Efficiency loss:

$$\begin{aligned} \Phi &= \frac{\sum_{t=1}^T f \left(\delta(t) - \sum_{i=1}^N s_i^c(t) \right)}{\sum_{t=1}^T f \left(\delta(t) - \sum_{i=1}^N s_i^g(t) \right)} \\ &= \frac{T \left(\frac{a}{2} \bar{\delta}^2 + b \bar{\delta} \right)}{\sum_{t=1}^T \frac{a}{2} \left(\frac{1}{N+1} \delta(t) + \frac{N}{N+1} \bar{\delta} \right)^2 + b \left(\frac{1}{N+1} \delta(t) + \frac{N}{N+1} \bar{\delta} \right)}. \end{aligned}$$

Letting $N \rightarrow \infty$, the efficiency loss vanishes, i.e. $\Phi \rightarrow 1$.

- “Price of anarchy”

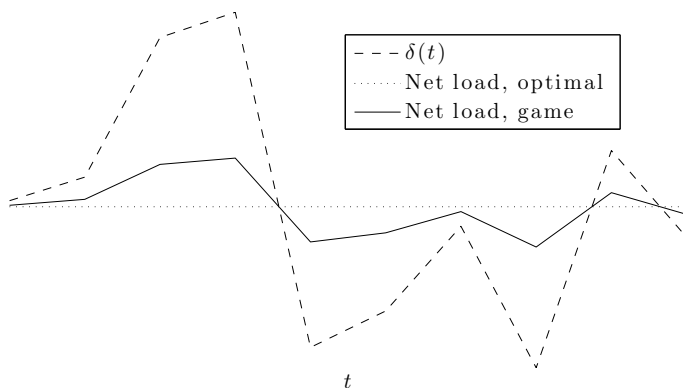


Figure 1: The nominal load without storage, δ , and the net load in the centralized and game outcomes with $N = 3$.

- Worst case - duopoly (only monopoly worse)
- Game shows variations allowed to persist to preserve arbitrage
- More participants flattens net load, approaches true optimum.

5 Bertrand-Edgeworth competition

- Firms submit prices, λ_i , $i = 1, 2$ (n player exists)
- Demand d
- Firm capacities: c_1, c_2
- Reservation utility: λ_r

Payoffs:

$$u_i(\lambda) = \begin{cases} \lambda_i \min\{c_i, d\}, & \lambda_i < \lambda_{-i} \\ \lambda_i \min\{c_i, (d - c_{-i})^+\}, & \lambda_i > \lambda_{-i} \end{cases}$$

Expected payoff:

$$\rho_i(m) = u_i(m) = \int_{0 \leq \lambda \leq \lambda_r} u_i(\lambda) m(\lambda) d\lambda = \int_{0 \leq \lambda \leq \lambda_r} m_1(\lambda_1) m_2(\lambda_2) u_i(\lambda)$$

- Case 1: $d \leq c_1, c_2$: Always left out player, PNE is $\lambda_i = 0$ and $\rho_i^*(c) = 0$. Standard Bertrand.
- Case 2: $d \geq c_1 + c_2$: Both fill up regardless, PNE is $\lambda_i = \lambda_r$, $\rho_i^*(c) = \lambda_r c_i$.

- Case 3: $c_j < d < c_1 + c_2$ for some j . An MNE exists in which player i 's profit is

$$\rho_i^*(c) = \frac{\lambda_r \min\{c_i, d\}}{\min\{c_{\max}, d\}} (d + c_{\max} - c_1 - c_2),$$

where $c_{\max} = \max_i c_i \dots$ holding sales.

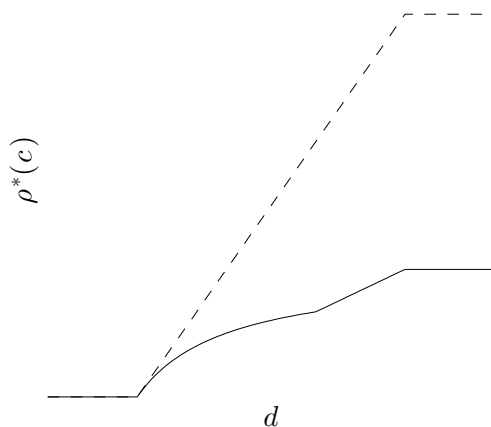


Figure 2: The expected profits of each player, $\rho_i^*(c)$, in a capacitated price duopoly for $\lambda_r = 1$, $c_1 = 1/5$, $c_2 = 3/5$, and $d \in [0, 1]$.

5.1 Application to transmission

- 2 nodes, same demand d
- Max price λ_r
- Generation p_1, p_2
- Line capacity \bar{s}
- Profits: $\lambda_i p_i$

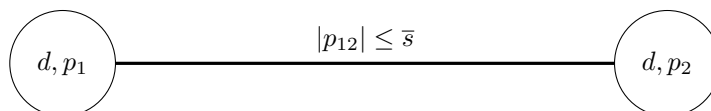


Figure 3: Market power in a congested two-node transmission network.

Payoff function:

$$\lambda_i \begin{cases} d + \min \{d, \bar{s}\} & \text{if } \lambda_i < \lambda_{-i} \\ \max \{d - \bar{s}, 0\} & \text{if } \lambda_i > \lambda_{-i} \\ d & \text{if } \lambda_i = \lambda_{-i} \end{cases} .$$

Equivalent to BE comp. with

- Demand $2d$
- capacities $d + \bar{s}$

Case-wise Analysis

- $d \leq \bar{s}$: Undercutting, PNE $\lambda_1 = \lambda_2 = 0$.
- $\bar{s} = 0$: Two monopolies, PNE $\lambda_i = \lambda_r$.
- $0 < \bar{s} < d$: No PNE. MNE:

$$\mu(\lambda_i) = \frac{\lambda_r (d - \bar{s})}{2\bar{s}\lambda_i^2}, \quad \lambda_i \in \left[\frac{\lambda_r (d - \bar{s})}{d + \bar{s}}, \lambda_r \right],$$

Expected profits: $\lambda_r (d - \bar{s})$.

6 Conclusion

- Game theory has limited tractability
- More market participants means better competition.
- Transmission congestion \rightarrow local monopolies (e.g., Enron)!
- Physics matter! Initial market designs ignored transmission.

References