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Technical University of Denmark, Lyngby, Denmark

EES-UETP Course title

Optimization problems with decomposable structure

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Learning Objectives

After this session the participants are expected to be able to:

- Explain the need for decomposition

- Identify whether each optimization problem is decomposable or not (if so, how?)

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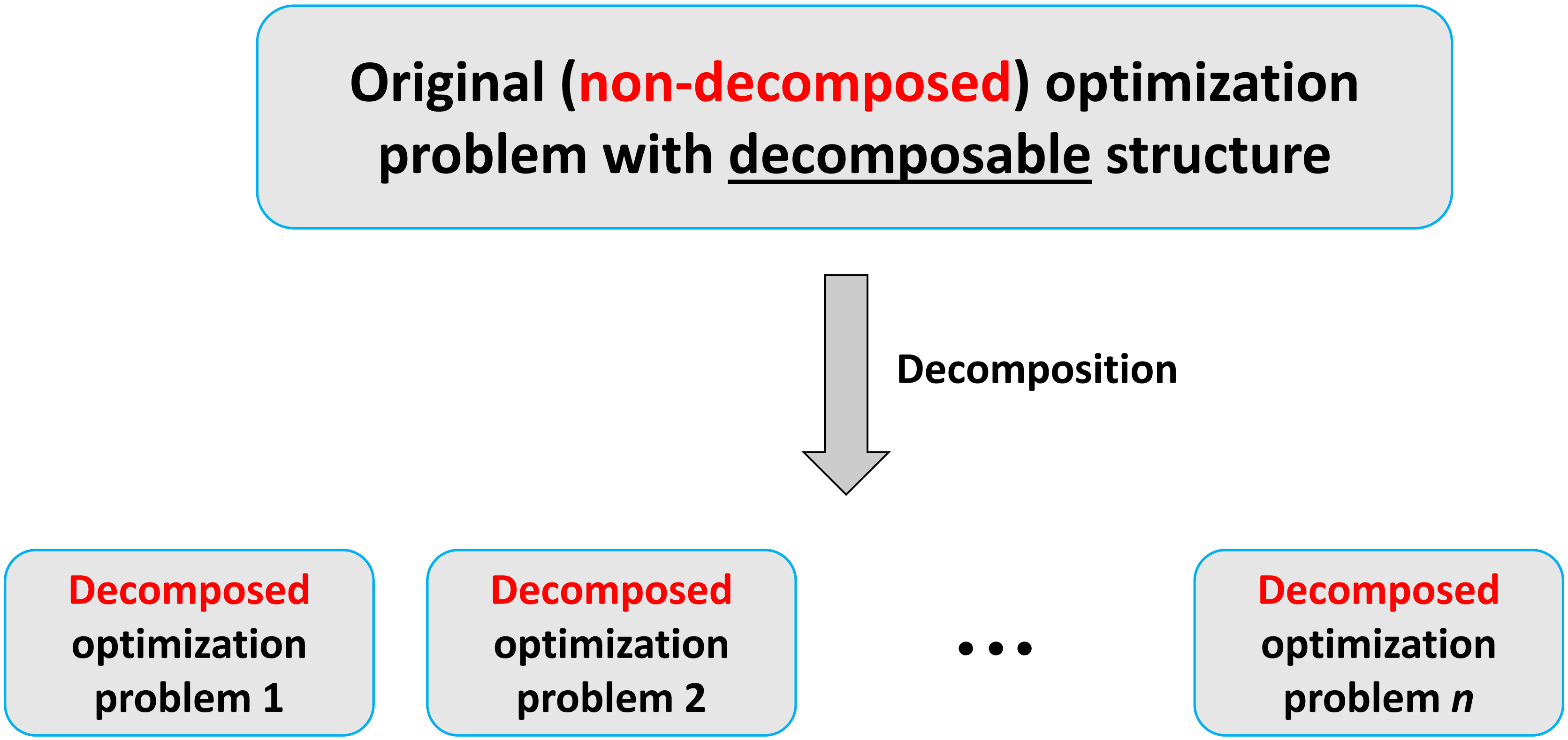
Decomposition

Main idea



Each decomposed problem is easier-to-solve than the original (non-decomposed) problem!

Decomposition

Motivation

- Why do we need decomposition in power systems?

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Decomposition

Motivation

- Why do we need decomposition in power systems?
  - Operation problems (e.g., unit commitment)
  - Planning problems (e.g., expansion)

Decomposition

Motivation

- Why do we need decomposition in power systems?
  - Operation problems (e.g., unit commitment)
    - Need to be computationally tractable
    - Need to be solved in a specific time period
  - Planning problems (e.g., investment)
    - Need to be computationally tractable

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Decomposition

Decomposable structure

Optimization problems with decomposable structure:

- Problems with complicating variable(s):

- Problems with complicating constraint(s):

Decomposition

Decomposable structure

Optimization problems with decomposable structure:

- Problems with complicating variable(s):

  The original problem is decomposed if the complicating variables are fixed to given values!

- Problems with complicating constraint(s):

  The original problem is decomposed if the complicating constraints are relaxed (removed)!

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Decomposition

Decomposable structure

Optimization problems with decomposable structure:

- Problems with complicating variable(s):

  The original problem is decomposed if the complicating variables are fixed to given values!

- Problems with complicating constraint(s):

  The original problem is decomposed if the complicating constraints are relaxed (removed)!

In the literature, "complicating" variables/constraints are also called as "coupling" variables/constraint!

Decomposition

Features of decomposition techniques

- Iterative solution techniques

- Original optimization problem decomposes to:
  - A single master problem (not always!)
  - A set of subproblems

Decomposable Structures

Optimization problems with complicating constraint(s)

Example: A linear programming (LP) as the original problem

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Example: A linear programming (LP) as the original problem

$$\underset{\substack{x_1,x_2,x_3,\\ y_1,y_2,\\ z_1,z_2,z_3}}{\text{Minimize}} \quad A_1x_1 + A_2x_2 + A_3x_3 + B_1y_1 + B_2y_2 + C_1z_1 + C_2z_2 + C_3z_3$$

Subject to

$$E_{11}x_1 + E_{12}x_2 + E_{13}x_3 \geq F_1$$
$$E_{21}x_1 + E_{22}x_2 + E_{23}x_3 \geq F_2$$

$$E_{31}y_1 + E_{32}y_2 \geq F_3$$

$$E_{41}z_1 + E_{42}z_2 + E_{43}z_3 \geq F_4$$
$$E_{51}z_1 + E_{52}z_2 + E_{53}z_3 \geq F_5$$

$$E_{61}x_1 + E_{62}x_2 + E_{63}x_3 + E_{64}y_1 + E_{65}y_2 + E_{66}z_1 + E_{67}z_2 + E_{68}z_3 \geq F_6$$

Decomposable Structures

Optimization problems with complicating constraint(s)

Example: A linear programming (LP) as the original problem

$$\underset{\substack{x_1,x_2,x_3,\\ y_1,y_2,\\ z_1,z_2,z_3}}{\text{Minimize}} \quad A_1x_1 + A_2x_2 + A_3x_3 + B_1y_1 + B_2y_2 + C_1z_1 + C_2z_2 + C_3z_3$$

Subject to

$$E_{11}x_1 + E_{12}x_2 + E_{13}x_3 \geq F_1$$
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Decomposable Structures

Optimization problems with complicating constraint(s)

Example: A linear programming (LP) as the original problem

$$\underset{\substack{x_1,x_2,x_3,\\ y_1,y_2,\\ z_1,z_2,z_3}}{\text{Minimize}} \quad A_1x_1 + A_2x_2 + A_3x_3 + B_1y_1 + B_2y_2 + C_1z_1 + C_2z_2 + C_3z_3$$

Subject to

Constraints including only variables x

$$E_{11}x_1 + E_{12}x_2 + E_{13}x_3 \geq F_1$$
$$E_{21}x_1 + E_{22}x_2 + E_{23}x_3 \geq F_2$$

Constraint including only variables y

$$E_{31}y_1 + E_{32}y_2 \geq F_3$$

Constraints including only variables z

$$E_{41}z_1 + E_{42}z_2 + E_{43}z_3 \geq F_4$$
$$E_{51}z_1 + E_{52}z_2 + E_{53}z_3 \geq F_5$$

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Decomposable Structures

Optimization problems with complicating constraint(s)

Example: A linear programming (LP) as the original problem

$$\underset{\substack{x_1,x_2,x_3,\\ y_1,y_2,\\ z_1,z_2,z_3}}{\text{Minimize}} \quad A_1x_1 + A_2x_2 + A_3x_3 + B_1y_1 + B_2y_2 + C_1z_1 + C_2z_2 + C_3z_3$$

Subject to

Constraints including only variables x

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Constraint including only variables y

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Decomposable Structures

Optimization problems with complicating constraint(s)

Example: A linear programming (LP) as the original problem

$$\underset{\substack{x_1,x_2,x_3,\\ y_1,y_2,\\ z_1,z_2,z_3}}{\text{Minimize}} \quad A_1x_1 + A_2x_2 + A_3x_3 + B_1y_1 + B_2y_2 + C_1z_1 + C_2z_2 + C_3z_3$$

Subject to

Constraints including only variables x

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This is complicating constraint: If relaxed (removed), then the original problem decomposes to three smaller optimization problems (subproblems)!

Decomposable Structures

Optimization problems with complicating constraint(s)

Example: A linear programming (LP) as the original problem

Subproblem 1:

$$\underset{x_1, x_2, x_3}{\text{Minimize}} \quad A_1 x_1 + A_2 x_2 + A_3 x_3$$

Subject to

Constraints including only variables x

$$E_{11} x_1 + E_{12} x_2 + E_{13} x_3 \geq F_1$$
$$E_{21} x_1 + E_{22} x_2 + E_{23} x_3 \geq F_2$$

Example: A linear programming (LP) as the original problem

Subproblem 2:

$$\underset{y_1, y_2}{\text{Minimize}} \quad B_1 y_1 + B_2 y_2$$

Subject to

Constraint including only variables y

$$E_{31} y_1 + E_{32} y_2 \geq F_3$$

Decomposable Structures

Optimization problems with complicating constraint(s)

Example: A linear programming (LP) as the original problem

Subproblem 3:

$$\underset{z_1, z_2, z_3}{\text{Minimize}} \quad C_1 z_1 + C_2 z_2 + C_3 z_3$$

Subject to

Constraints including only variables z

$$E_{41} z_1 + E_{42} z_2 + E_{43} z_3 \geq F_4$$
$$E_{51} z_1 + E_{52} z_2 + E_{53} z_3 \geq F_5$$

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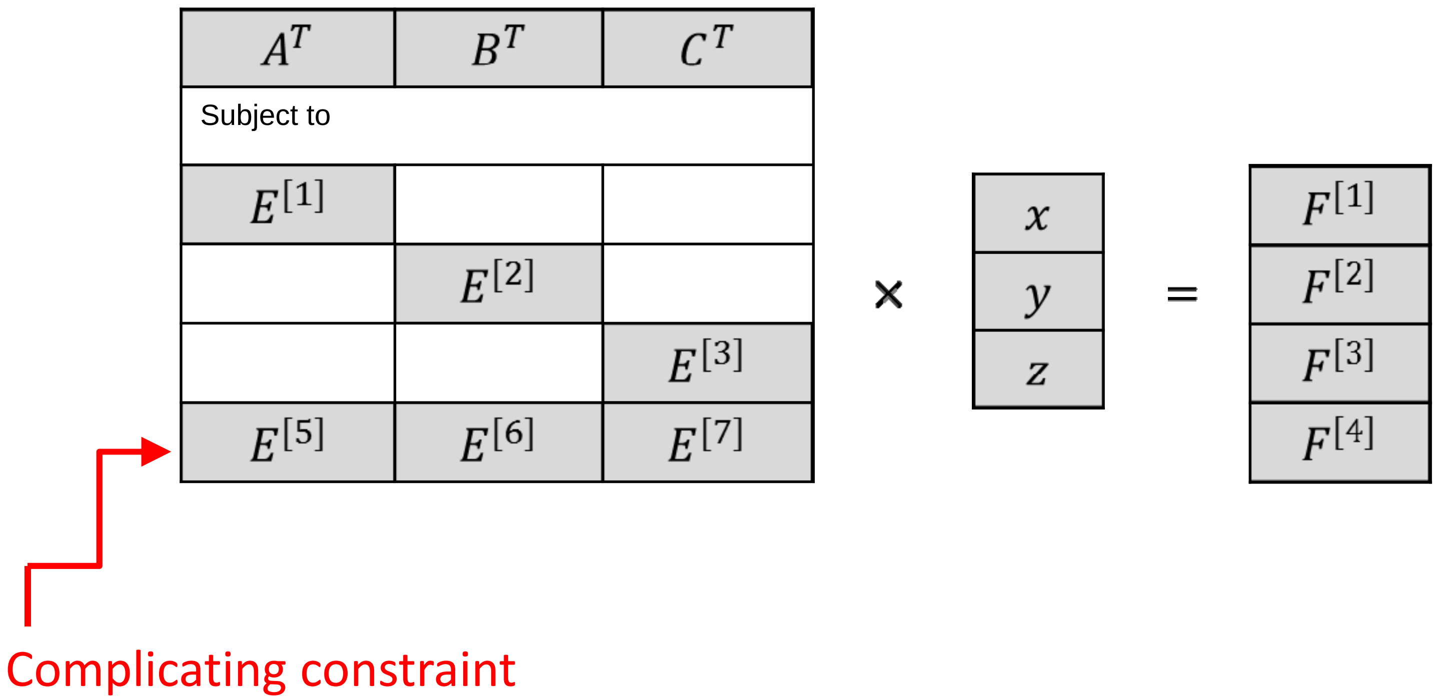
Decomposable Structures

Optimization problems with complicating constraint(s)

Example: A linear programming (LP) as the original problem

Decomposable matrix with complicating constraint(s)



Example: A linear programming (LP) as the original problem

Example: A linear programming (LP) as the original problem

$$\underset{\substack{x_1,x_2,\\ y_1,y_2,\\ z_1,z_2,\\ \beta}}{\text{Minimize}} \quad A_1x_1 + A_2x_2 + B_1y_1 + B_2y_2 + C_1z_1 + C_2z_2 + D_1\,\beta$$

Subject to

$$E_{11}x_1 + E_{12}\,x_2 + E_{13}\,\beta \geq F_1$$
$$E_{21}x_1 + E_{22}\,x_2 + E_{23}\beta \geq F_2$$

$$E_{31}y_1 + E_{32}\,y_2 + E_{33}\,\beta \geq F_3$$

$$E_{41}z_1 + E_{42}\,z_2 + E_{43}\,\beta \geq F_4$$
$$E_{51}z_1 + E_{52}\,z_2 + E_{53}\,\beta \geq F_5$$

Decomposable Structures

Optimization problems with complicating variable(s)

Example: A linear programming (LP) as the original problem

$$\underset{x_1,x_2,\; y_1,y_2,\; z_1,z_2,\; \beta}{\text{Minimize}} \quad A_1 x_1 + A_2 x_2 + B_1 y_1 + B_2 y_2 + C_1 z_1 + C_2 z_2 + D_1 \beta$$

Subject to

$$E_{11} x_1 + E_{12} x_2 + E_{13} \beta \geq F_1$$
$$E_{21} x_1 + E_{22} x_2 + E_{23} \beta \geq F_2$$

$$E_{31} y_1 + E_{32} y_2 + E_{33} \beta \geq F_3$$

$$E_{41} z_1 + E_{42} z_2 + E_{43} \beta \geq F_4$$
$$E_{51} z_1 + E_{52} z_2 + E_{53} \beta \geq F_5$$

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Decomposable Structures

Optimization problems with complicating variable(s)

Example: A linear programming (LP) as the original problem

$$\underset{\substack{x_1,x_2,\\ y_1,y_2,\\ z_1,z_2,\\ \beta}}{\text{Minimize}} \quad A_1 x_1 + A_2 x_2 + B_1 y_1 + B_2 y_2 + C_1 z_1 + C_2 z_2 + D_1 \beta$$

Subject to

$$E_{11} x_1 + E_{12} x_2 + E_{13} \beta \geq F_1$$
$$E_{21} x_1 + E_{22} x_2 + E_{23} \beta \geq F_2$$

$$E_{31} y_1 + E_{32} y_2 + E_{33} \beta \geq F_3$$

$$E_{41} z_1 + E_{42} z_2 + E_{43} \beta \geq F_4$$
$$E_{51} z_1 + E_{52} z_2 + E_{53} \beta \geq F_5$$

β is a complicating variable, i.e., if it is fixed to a given value (β^{fixed}), then the original problem decomposes to 3 smaller problems (subproblems)!

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Decomposable Structures

Optimization problems with complicating variable(s)

Example: A linear programming (LP) as the original problem

$$\underset{\substack{x_1,x_2,\\ y_1,y_2,\\ z_1,z_2,\\ \beta}}{\text{Minimize}} \quad A_1 x_1 + A_2 x_2 + B_1 y_1 + B_2 y_2 + C_1 z_1 + C_2 z_2 + \underbrace{D_1\, \beta^{\text{fixed}}}$$

Fixed term, it can be removed from objective function

Subject to

Constraints including only variables x

$$E_{11} x_1 + E_{12} x_2 \geq F_1 - E_{13}\, \beta^{\text{fixed}}$$
$$E_{21} x_1 + E_{22} x_2 \geq F_2 - E_{23}\, \beta^{\text{fixed}}$$

Constraint including only variables y

$$E_{31} y_1 + E_{32} y_2 \geq F_3 - E_{33}\, \beta^{\text{fixed}}$$

Constraints including only variables z

$$E_{41} z_1 + E_{42} z_2 \geq F_4 - E_{43}\, \beta^{\text{fixed}}$$
$$E_{51} z_1 + E_{52} z_2 \geq F_5 - E_{53}\, \beta^{\text{fixed}}$$

β is a complicating variable, i.e., if it is fixed to a given value (β^{fixed}), then the original problem decomposes to 3 smaller problems (subproblems)!

Decomposable Structures

Optimization problems with complicating variable(s)

Example: A linear programming (LP) as the original problem

Subproblem 1:

$$\underset{x_1, x_2}{\text{Minimize}} \quad A_1 x_1 + A_2 x_2$$

Subject to

Constraints including only variables x

$$E_{11} x_1 + E_{12} x_2 \geq F_1 - E_{13}\, \beta^{\text{fixed}}$$
$$E_{21} x_1 + E_{22} x_2 \geq F_2 - E_{23}\, \beta^{\text{fixed}}$$

Example: A linear programming (LP) as the original problem

Subproblem 2:

$$\underset{y_1, y_2}{\text{Minimize}}\ B_1 y_1 + B_2 y_2$$

Subject to

Constraint including only variables y

$$E_{31} y_1 + E_{32} y_2 \geq F_3 - E_{33}\ \beta^{\text{fixed}}$$

Example: A linear programming (LP) as the original problem

Subproblem 3:

$$\underset{z_1, z_2}{\text{Minimize}}\ C_1 z_1 + C_2 z_2$$

Subject to

Constraints including only variables z

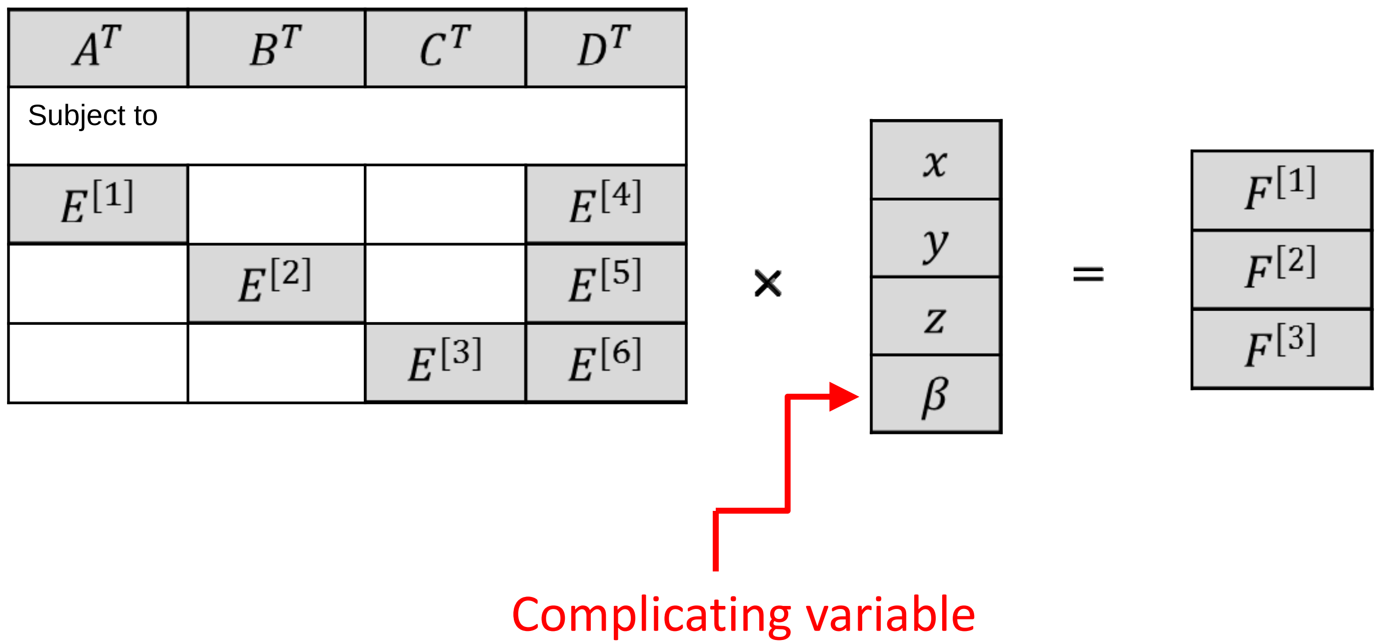
$$E_{41} z_1 + E_{42} z_2 \geq F_4 - E_{43}\ \beta^{\text{fixed}}$$
$$E_{51} z_1 + E_{52} z_2 \geq F_5 - E_{53}\ \beta^{\text{fixed}}$$

Decomposable Structures

Optimization problems with complicating variable(s)

Example: A linear programming (LP) as the original problem

Decomposable matrix with complicating variable(s)



Exercise:

Seven examples are available in the papers on your table. Please check them in the next 15 minutes, and identify whether they are decomposable problems or not (if so, how?). Then, check your results with your neighbors around the table.

Example 1

Identify whether the following problem is decomposable or not (if so, how)!

$$\underset{x_1,x_2,x_3,y_1,y_2,y_3,z_1,z_2,z_3,w_1}{\text{Minimize}} \quad -4x_1 - y_1 - 6z_1$$

subject to

$$x_1 - x_2 = 1$$

$$x_1 + x_3 = 3$$

$$y_1 - y_2 = 1$$

$$y_1 + y_3 = 2$$

$$z_1 - z_2 = 1$$

$$z_1 + z_3 = 2$$

$$3x_1 + 2y_1 + 4z_1 + w_1 = 17$$

$$x_1, x_2, x_3, y_1, y_2, y_3, z_1, z_2, z_3, w_1 \geq 0$$

Example 1

Identify whether the following problem is decomposable or not (if so, how)!

$$\underset{x_1,x_2,x_3,y_1,y_2,y_3,z_1,z_2,z_3,w_1}{\text{Minimize}} \quad -4x_1 - y_1 - 6z_1$$

subject to

$$x_1 - x_2 = 1$$

$$x_1 + x_3 = 3$$

$$y_1 - y_2 = 1$$

$$y_1 + y_3 = 2$$

$$z_1 - z_2 = 1$$

$$z_1 + z_3 = 2$$

$$\boxed{3x_1 + 2y_1 + 4z_1 + w_1 = 17}$$

$$x_1, x_2, x_3, y_1, y_2, y_3, z_1, z_2, z_3, w_1 \geq 0$$

Complicating constraint:
if relaxed, then the original problem decomposes to 3 subproblems

Example 2

Identify whether the following problem is decomposable or not (if so, how)!

$$\underset{x_1,x_2,x_3,x_4,x_5,x_6}{\text{Maximize}} \; 4x_1 + 3x_2 + 2x_3 + 3x_4 + 2x_5 + 3x_6$$

subject to

$$x_1 - 2x_2 + 2x_6 \leq 3$$

$$2x_1 + x_2 + x_6 \leq 3$$

$$-2x_1 + 3x_2 + x_6 \leq 7$$

$$x_3 + 3x_6 \leq 4$$

$$2x_3 - x_6 \leq 3$$

$$x_4 \leq 1$$

$$2x_4 - 4x_5 + 3x_6 \leq 5$$

$$3x_4 + x_5 - x_6 \leq 4$$

Example 2

Identify whether the following problem is decomposable or not (if so, how)!

$$\underset{x_1,x_2,x_3,x_4,x_5,x_6}{\text{Maximize}} \; 4x_1 + 3x_2 + 2x_3 + 3x_4 + 2x_5 + 3x_6$$

subject to

$$x_1 - 2x_2 + 2x_6 \leq 3$$

$$2x_1 + x_2 + x_6 \leq 3$$

$$-2x_1 + 3x_2 + x_6 \leq 7$$

$$x_3 + 3x_6 \leq 4$$

$$2x_3 - x_6 \leq 3$$

$$x_4 \leq 1$$

$$2x_4 - 4x_5 + 3x_6 \leq 5$$

$$3x_4 + x_5 - x_6 \leq 4$$

x_6 is a complicating variable:
if fixed to a given value, then the original problem decomposes to 3 subproblems

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Example 3

Identify whether the following problem is decomposable or not (if so, how)!

Single-node single-hour optimal power flow (OPF) problem (total cost minimization) with 3 conventional generators and a single inelastic load:

$$\underset{g_1, g_2, g_3}{\text{Minimize}} \quad 10g_1 + 25g_2 + 30g_3$$

subject to

$$0 \leq g_1 \leq 100$$

$$0 \leq g_2 \leq 150$$

$$0 \leq g_3 \leq 200$$

$$g_1 + g_2 + g_3 = 350$$

Example 3

Identify whether the following problem is decomposable or not (if so, how)!

Single-node single-hour optimal power flow (OPF) problem (total cost minimization) with 3 conventional generators and a single inelastic load:

$$\underset{g_1, g_2, g_3}{\text{Minimize}} \quad 10g_1 + 25g_2 + 30g_3$$

subject to

$$0 \leq g_1 \leq 100$$

$$0 \leq g_2 \leq 150$$

$$0 \leq g_3 \leq 200$$

$$\boxed{g_1 + g_2 + g_3 = 350}$$

→ **Complicating constraint:** if relaxed, then the original problem decomposes to 3 subproblems, <u>one per generator</u>

Example 4

Identify whether the following problem is decomposable or not (if so, how)!

Single-node single-hour optimal power flow (OPF) problem (social welfare maximization) with 3 conventional generators and a single elastic load:

$$\underset{g_1,g_2,g_3,d_1}{\text{Maximize}} \; 40d_1 - 10g_1 - 25g_2 - 30g_3$$

subject to

$$0 \le g_1 \le 100$$

$$0 \le g_2 \le 150$$

$$0 \le g_3 \le 200$$

$$0 \le d_1 \le 350$$

$$g_1 + g_2 + g_3 = d_1$$

Example 4

Identify whether the following problem is decomposable or not (if so, how)!

Single-node single-hour optimal power flow (OPF) problem (social welfare maximization) with 3 conventional generators and a single elastic load:

$$\underset{g_1,g_2,g_3,d_1}{\text{Maximize}}\ 40d_1 - 10g_1 - 25g_2 - 30g_3$$

subject to

$$0 \le g_1 \le 100$$

$$0 \le g_2 \le 150$$

$$0 \le g_3 \le 200$$

$$0 \le d_1 \le 350$$

$$g_1 + g_2 + g_3 = d_1$$

Complicating constraint:
if relaxed, then the original problem decomposes to 4 subproblems, one per agent (generator and demand)

Example 5

Identify whether the following problem is decomposable or not (if so, how)!

Single-node multi-hour (index: h) optimal power flow (OPF) problem (social welfare maximization) with 3 conventional generators and a single elastic load:

$$\underset{g_{h,1},g_{h,2},g_{h,3},d_{h,1}}{\text{Maximize}} \quad \sum_{h}\left[40d_{h,1} - 10g_{h,1} - 25g_{h,2} - 30g_{h,3}\right]$$

subject to

$$0 \le g_{h,1} \le 100 \quad \forall h$$

$$0 \le g_{h,2} \le 150 \quad \forall h$$

$$0 \le g_{h,3} \le 200 \quad \forall h$$

$$0 \le d_{h,1} \le D_{h,1} \quad \forall h$$

$$g_{h,1} + g_{h,2} + g_{h,3} = d_{h,1} \quad \forall h$$

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Example 5

Identify whether the following problem is decomposable or not (if so, how)!

Single-node multi-hour (index: h) optimal power flow (OPF) problem (social welfare maximization) with 3 conventional generators and a single elastic load:

$$\underset{g_{h,1}, g_{h,2}, g_{h,3}, d_{h,1}}{\text{Maximize}} \sum_{h} [40 d_{h,1} - 10 g_{h,1} - 25 g_{h,2} - 30 g_{h,3}]$$

subject to

$$0 \leq g_{h,1} \leq 100 \quad \forall h$$

$$0 \leq g_{h,2} \leq 150 \quad \forall h$$

$$0 \leq g_{h,3} \leq 200 \quad \forall h$$

$$0 \leq d_{h,1} \leq D_{h,1} \quad \forall h$$

$$g_{h,1} + g_{h,2} + g_{h,3} = d_{h,1} \quad \forall h$$

Complicating constraints: if relaxed, then the original problem decomposes to a set of subproblems, <u>one per agent per hour</u>

Example 5

Identify whether the following problem is decomposable or not (if so, how)!

Single-node multi-hour (index: h) optimal power flow (OPF) problem (social welfare maximization) with 3 conventional generators and a single elastic load:

$$\underset{g_{h,1},g_{h,2},g_{h,3},d_{h,1}}{\text{Maximize}} \sum_{h} [40d_{h,1} - 10g_{h,1} - 25g_{h,2} - 30g_{h,3}]$$

subject to

$$0 \leq g_{h,1} \leq 100 \quad \forall h$$

$$0 \leq g_{h,2} \leq 150 \quad \forall h$$

$$0 \leq g_{h,3} \leq 200 \quad \forall h$$

$$0 \leq d_{h,1} \leq D_{h,1} \quad \forall h$$

$$\boxed{g_{h,1} + g_{h,2} + g_{h,3} = d_{h,1} \quad \forall h}$$

- **Number of complicating constraints: ?**
- **Number of subproblem: ?**

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Example 5

Identify whether the following problem is decomposable or not (if so, how)!

Single-node multi-hour (index: h) optimal power flow (OPF) problem (social welfare maximization) with 3 conventional generators and a single elastic load:

$$\underset{g_{h,1},g_{h,2},g_{h,3},d_{h,1}}{\text{Maximize}} \quad \sum_{h} [40d_{h,1} - 10g_{h,1} - 25g_{h,2} - 30g_{h,3}]$$

subject to

$$0 \leq g_{h,1} \leq 100 \quad \forall h$$

$$0 \leq g_{h,2} \leq 150 \quad \forall h$$

$$0 \leq g_{h,3} \leq 200 \quad \forall h$$

$$0 \leq d_{h,1} \leq D_{h,1} \quad \forall h$$

$$\boxed{g_{h,1} + g_{h,2} + g_{h,3} = d_{h,1} \quad \forall h}$$

- **Number of complicating constraints: |h|**
- **Number of subproblem: 4|h|**

Example 6

Identify whether the following problem is decomposable or not (if so, how)!

Single-node multi-hour (index: h) optimal power flow (OPF) problem (social welfare maximization) with 3 conventional generators and a single elastic load (enforcing inter-temporal ramping constraints for one of generators):

$$\underset{g_{h,1}, g_{h,2}, g_{h,3}, d_{h,1}}{\text{Maximize}} \sum_h [40 d_{h,1} - 10 g_{h,1} - 25 g_{h,2} - 30 g_{h,3}]$$

subject to

$$0 \leq g_{h,1} \leq 100 \quad \forall h$$

$$0 \leq g_{h,2} \leq 150 \quad \forall h$$

$$0 \leq g_{h,3} \leq 200 \quad \forall h$$

$$0 \leq d_{h,1} \leq D_{h,1} \quad \forall h$$

$$-25 \leq [g_{h,1} - g_{(h-1),1}] \leq 25 \quad \forall h$$

$$g_{h,1} + g_{h,2} + g_{h,3} = d_{h,1} \quad \forall h$$

Example 6

Identify whether the following problem is decomposable or not (if so, how)!

Single-node multi-hour (index: h) optimal power flow (OPF) problem (social welfare maximization) with 3 conventional generators and a single elastic load (enforcing inter-temporal ramping constraints for one of generators):

$$\underset{g_{h,1},g_{h,2},g_{h,3},d_{h,1}}{\text{Maximize}} \quad \sum_{h}\left[40d_{h,1} - 10g_{h,1} - 25g_{h,2} - 30g_{h,3}\right]$$

subject to

$$0 \le g_{h,1} \le 100 \quad \forall h$$

$$0 \le g_{h,2} \le 150 \quad \forall h$$

$$0 \le g_{h,3} \le 200 \quad \forall h$$

$$0 \le d_{h,1} \le D_{h,1} \quad \forall h$$

$$-25 \le \left[g_{h,1} - g_{(h-1),1}\right] \le 25 \quad \forall h$$

$$g_{h,1} + g_{h,2} + g_{h,3} = d_{h,1} \quad \forall h$$

Investigate the following three options:

1) relax ramping constraints only,
2) relax balance constraints only,
3) relax both.

Example 6

Identify whether the following problem is decomposable or not (if so, how)!

Single-node multi-hour (index: h) optimal power flow (OPF) problem (social welfare maximization) with 3 conventional generators and a single elastic load (enforcing inter-temporal ramping constraints for one of generators):

$$\underset{g_{h,1},g_{h,2},g_{h,3},d_{h,1}}{\text{Maximize}} \sum_{h} [40d_{h,1} - 10g_{h,1} - 25g_{h,2} - 30g_{h,3}]$$

subject to

$$0 \le g_{h,1} \le 100 \quad \forall h$$

$$0 \le g_{h,2} \le 150 \quad \forall h$$

$$0 \le g_{h,3} \le 200 \quad \forall h$$

$$0 \le d_{h,1} \le D_{h,1} \quad \forall h$$

$$-25 \le [g_{h,1} - g_{(h-1),1}] \le 25 \quad \forall h$$

$$g_{h,1} + g_{h,2} + g_{h,3} = d_{h,1} \quad \forall h$$

Option 1:

- Number of complicating constraints: ?
- Number of subproblem: ?

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Example 6

Identify whether the following problem is decomposable or not (if so, how)!

Single-node multi-hour (index: h) optimal power flow (OPF) problem (social welfare maximization) with 3 conventional generators and a single elastic load (enforcing inter-temporal ramping constraints for one of generators):

$$\underset{g_{h,1}, g_{h,2}, g_{h,3}, d_{h,1}}{\text{Maximize}} \sum_{h} [40 d_{h,1} - 10 g_{h,1} - 25 g_{h,2} - 30 g_{h,3}]$$

subject to

$$0 \leq g_{h,1} \leq 100 \quad \forall h$$

$$0 \leq g_{h,2} \leq 150 \quad \forall h$$

$$0 \leq g_{h,3} \leq 200 \quad \forall h$$

$$0 \leq d_{h,1} \leq D_{h,1} \quad \forall h$$

$$-25 \leq [g_{h,1} - g_{(h-1),1}] \leq 25 \quad \forall h$$

$$g_{h,1} + g_{h,2} + g_{h,3} = d_{h,1} \quad \forall h$$

Option 1:

- Number of complicating constraints: |h|
- Number of subproblem: |h|

Example 6

Identify whether the following problem is decomposable or not (if so, how)!

Single-node multi-hour (index: h) optimal power flow (OPF) problem (social welfare maximization) with 3 conventional generators and a single elastic load (enforcing inter-temporal ramping constraints for one of generators):

$$\underset{g_{h,1},g_{h,2},g_{h,3},d_{h,1}}{\text{Maximize}} \quad \sum_{h}[40d_{h,1} - 10g_{h,1} - 25g_{h,2} - 30g_{h,3}]$$

subject to

$$0 \leq g_{h,1} \leq 100 \quad \forall h$$

$$0 \leq g_{h,2} \leq 150 \quad \forall h$$

$$0 \leq g_{h,3} \leq 200 \quad \forall h$$

$$0 \leq d_{h,1} \leq D_{h,1} \quad \forall h$$

$$-25 \leq [g_{h,1} - g_{(h-1),1}] \leq 25 \quad \forall h$$

$$g_{h,1} + g_{h,2} + g_{h,3} = d_{h,1} \quad \forall h$$

Option 2:

- Number of complicating constraints: ?
- Number of subproblem: ?

Example 6

Identify whether the following problem is decomposable or not (if so, how)!

Single-node multi-hour (index: h) optimal power flow (OPF) problem (social welfare maximization) with 3 conventional generators and a single elastic load (enforcing inter-temporal ramping constraints for one of generators):

$$\underset{g_{h,1},g_{h,2},g_{h,3},d_{h,1}}{\text{Maximize}} \sum_{h} [40d_{h,1} - 10g_{h,1} - 25g_{h,2} - 30g_{h,3}]$$

subject to

$$0 \leq g_{h,1} \leq 100 \quad \forall h$$

$$0 \leq g_{h,2} \leq 150 \quad \forall h$$

$$0 \leq g_{h,3} \leq 200 \quad \forall h$$

$$0 \leq d_{h,1} \leq D_{h,1} \quad \forall h$$

$$-25 \leq [g_{h,1} - g_{(h-1),1}] \leq 25 \quad \forall h$$

$$g_{h,1} + g_{h,2} + g_{h,3} = d_{h,1} \quad \forall h$$

Option 2:

- Number of complicating constraints: |h|
- Number of subproblem: 3|h|+1

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Example 6

Identify whether the following problem is decomposable or not (if so, how)!

Single-node multi-hour (index: h) optimal power flow (OPF) problem (social welfare maximization) with 3 conventional generators and a single elastic load (enforcing inter-temporal ramping constraints for one of generators):

$$\underset{g_{h,1},g_{h,2},g_{h,3},d_{h,1}}{\text{Maximize}} \sum_{h} [40d_{h,1} - 10g_{h,1} - 25g_{h,2} - 30g_{h,3}]$$

subject to

$$0 \le g_{h,1} \le 100 \quad \forall h$$

$$0 \le g_{h,2} \le 150 \quad \forall h$$

$$0 \le g_{h,3} \le 200 \quad \forall h$$

$$0 \le d_{h,1} \le D_{h,1} \quad \forall h$$

$$-25 \le [g_{h,1} - g_{(h-1),1}] \le 25 \quad \forall h$$

$$g_{h,1} + g_{h,2} + g_{h,3} = d_{h,1} \quad \forall h$$

Option 3:

- Number of complicating constraints: ?
- Number of subproblem: ?

Example 6

Identify whether the following problem is decomposable or not (if so, how)!

Single-node multi-hour (index: h) optimal power flow (OPF) problem (social welfare maximization) with 3 conventional generators and a single elastic load (enforcing inter-temporal ramping constraints for one of generators):

$$\underset{g_{h,1}, g_{h,2}, g_{h,3}, d_{h,1}}{\text{Maximize}} \sum_{h} [40 d_{h,1} - 10 g_{h,1} - 25 g_{h,2} - 30 g_{h,3}]$$

subject to

$$0 \leq g_{h,1} \leq 100 \quad \forall h$$

$$0 \leq g_{h,2} \leq 150 \quad \forall h$$

$$0 \leq g_{h,3} \leq 200 \quad \forall h$$

$$0 \leq d_{h,1} \leq D_{h,1} \quad \forall h$$

$$-25 \leq [g_{h,1} - g_{(h-1),1}] \leq 25 \quad \forall h$$

$$g_{h,1} + g_{h,2} + g_{h,3} = d_{h,1} \quad \forall h$$

Option 3:

- Number of complicating constraints: 2|h|
- Number of subproblem: 4|h|

Example 7

Identify whether the following problem is decomposable or not (if so, how)!

Single-node single-year (static) generation expansion problem (GEP), considering two existing generators (units 1 and 2) and one candidate generator (unit 3)

$$\underset{g_{h,1},g_{h,2},g_{h,3},x_3}{\text{Minimize}} \quad 15000x_3 + \sum_h \left[10g_{h,1} + 25g_{h,2} + 30g_{h,3}\right]$$

subject to

$$0 \leq g_{h,1} \leq 100 \quad \forall h$$

$$0 \leq g_{h,2} \leq 150 \quad \forall h$$

$$0 \leq g_{h,3} \leq x_3 \quad \forall h$$

$$g_{h,1} + g_{h,2} + g_{h,3} = D_{h,1} \quad \forall h$$

$$x_3 \geq 0$$

Example 7

Identify whether the following problem is decomposable or not (if so, how)!

Single-node single-year (static) generation expansion problem (GEP), considering two existing generators (units 1 and 2) and one candidate generator (unit 3)

$$\underset{g_{h,1},g_{h,2},g_{h,3},x_3}{\text{Minimize}} \quad 15000x_3 + \sum_h \left[10g_{h,1} + 25g_{h,2} + 30g_{h,3}\right]$$

subject to

$$0 \leq g_{h,1} \leq 100 \quad \forall h$$

$$0 \leq g_{h,2} \leq 150 \quad \forall h$$

$$0 \leq g_{h,3} \leq x_3 \quad \forall h$$

$$g_{h,1} + g_{h,2} + g_{h,3} = D_{h,1} \quad \forall h$$

$$x_3 \geq 0$$

Investigate the following two options:

1) x_3 is a complicating **variable**,
2) Balance conditions are complicating **constraints**.

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Example 7

Identify whether the following problem is decomposable or not (if so, how)!

Single-node single-year (static) generation expansion problem (GEP), considering two existing generators (units 1 and 2) and one candidate generator (unit 3)

$$\underset{g_{h,1},g_{h,2},g_{h,3},x_3}{\text{Minimize}} \quad 15000x_3 + \sum_h [10g_{h,1} + 25g_{h,2} + 30g_{h,3}]$$

subject to

$$0 \leq g_{h,1} \leq 100 \quad \forall h$$

$$0 \leq g_{h,2} \leq 150 \quad \forall h$$

$$0 \leq g_{h,3} \leq x_3 \quad \forall h$$

$$g_{h,1} + g_{h,2} + g_{h,3} = D_{h,1} \quad \forall h$$

$$x_3 \geq 0$$

Option 1 (fixing x_3):

- Number of complicating variables: ?
- Number of subproblem: ?

Example 7

Identify whether the following problem is decomposable or not (if so, how)!

Single-node single-year (static) generation expansion problem (GEP), considering two existing generators (units 1 and 2) and one candidate generator (unit 3)

$$\underset{g_{h,1},g_{h,2},g_{h,3},x_3}{\text{Minimize}} \quad 15000x_3 + \sum_h \left[10g_{h,1} + 25g_{h,2} + 30g_{h,3}\right]$$

subject to

$$0 \leq g_{h,1} \leq 100 \quad \forall h$$

$$0 \leq g_{h,2} \leq 150 \quad \forall h$$

$$0 \leq g_{h,3} \leq x_3 \quad \forall h$$

$$g_{h,1} + g_{h,2} + g_{h,3} = D_{h,1} \quad \forall h$$

$$x_3 \geq 0$$

Option 1 (fixing x_3):

- Number of complicating variables: 1
- Number of subproblem: |h|

Example 7

Identify whether the following problem is decomposable or not (if so, how)!

Single-node single-year (static) generation expansion problem (GEP), considering two existing generators (units 1 and 2) and one candidate generator (unit 3)

$$\underset{g_{h,1},g_{h,2},g_{h,3},x_3}{\text{Minimize}} \quad 15000x_3 + \sum_h [10g_{h,1} + 25g_{h,2} + 30g_{h,3}]$$

subject to

$$0 \leq g_{h,1} \leq 100 \quad \forall h$$

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$$0 \leq g_{h,3} \leq x_3 \quad \forall h$$

$$g_{h,1} + g_{h,2} + g_{h,3} = D_{h,1} \quad \forall h$$

$$x_3 \geq 0$$

Option 2 (relaxing balance constraints):

- Number of complicating constraints: ?
- Number of subproblem: ?

Example 7

Identify whether the following problem is decomposable or not (if so, how)!

Single-node single-year (static) generation expansion problem (GEP), considering two existing generators (units 1 and 2) and one candidate generator (unit 3)

$$\underset{g_{h,1},g_{h,2},g_{h,3},x_3}{\text{Minimize}} \quad 15000x_3 + \sum_h \left[10g_{h,1} + 25g_{h,2} + 30g_{h,3}\right]$$

subject to

$$0 \leq g_{h,1} \leq 100 \quad \forall h$$

$$0 \leq g_{h,2} \leq 150 \quad \forall h$$

$$0 \leq g_{h,3} \leq x_3 \quad \forall h$$

$$g_{h,1} + g_{h,2} + g_{h,3} = D_{h,1} \quad \forall h$$

$$x_3 \geq 0$$

Option 2 (relaxing balance constraints):

- Number of complicating constraints: |h|
- Number of subproblem: 2|h|+1

Reference

A. J. Conejo, E. Castillo, R. Minguez, and R. Garcia-Bertrand, *Decomposition Techniques in Mathematical Programming: Engineering and Science Applications*. Berlin, Germany: Springer, 2006.

Thanks for your attention!

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