Decomposition techniques for optimization problems with complicating variables

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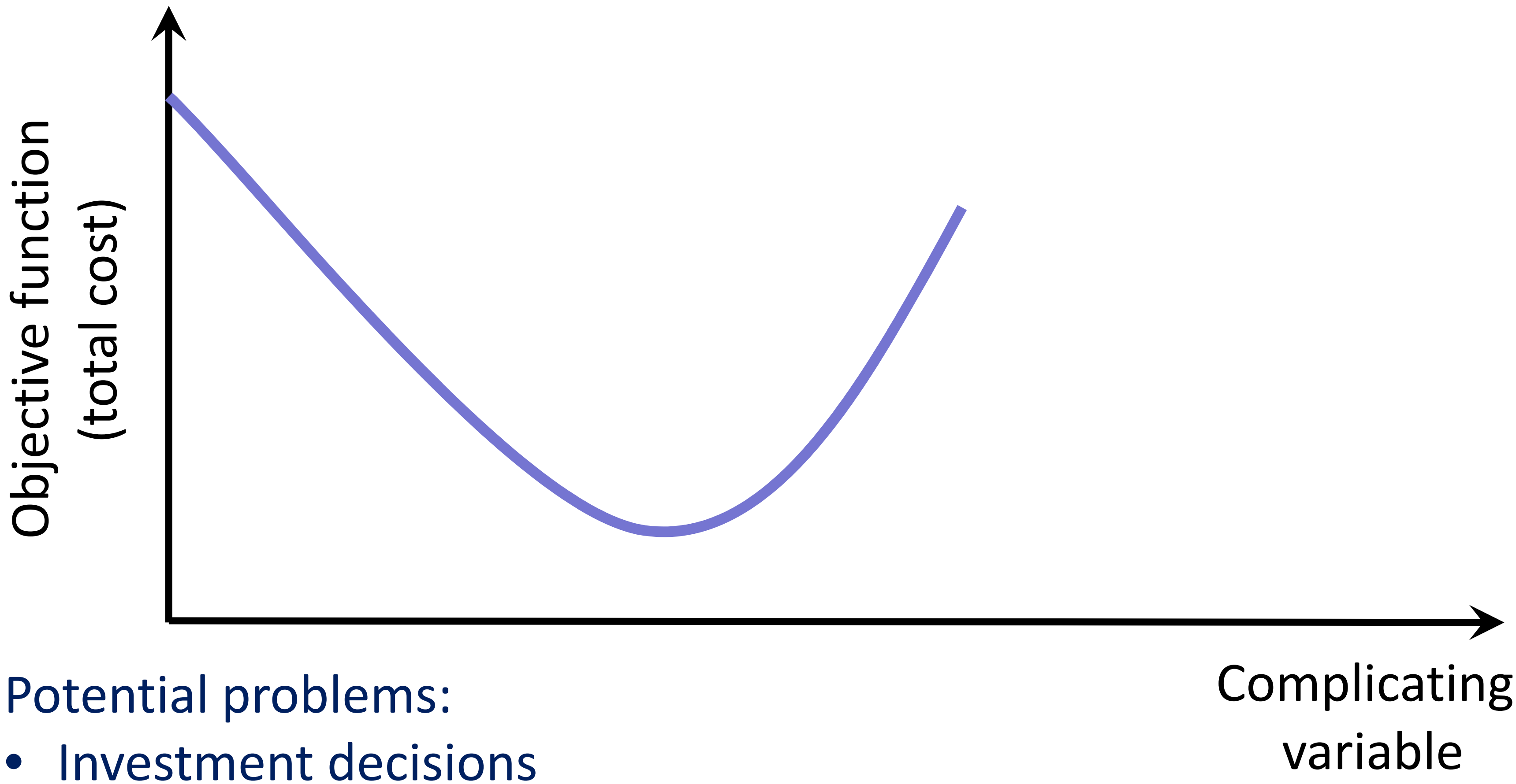
Learning Objectives

After this session the participants are expected to be able to:

- Explain the functioning of Benders' decomposition
- Explain the application of Benders' decomposition to multi-stage stochastic problems

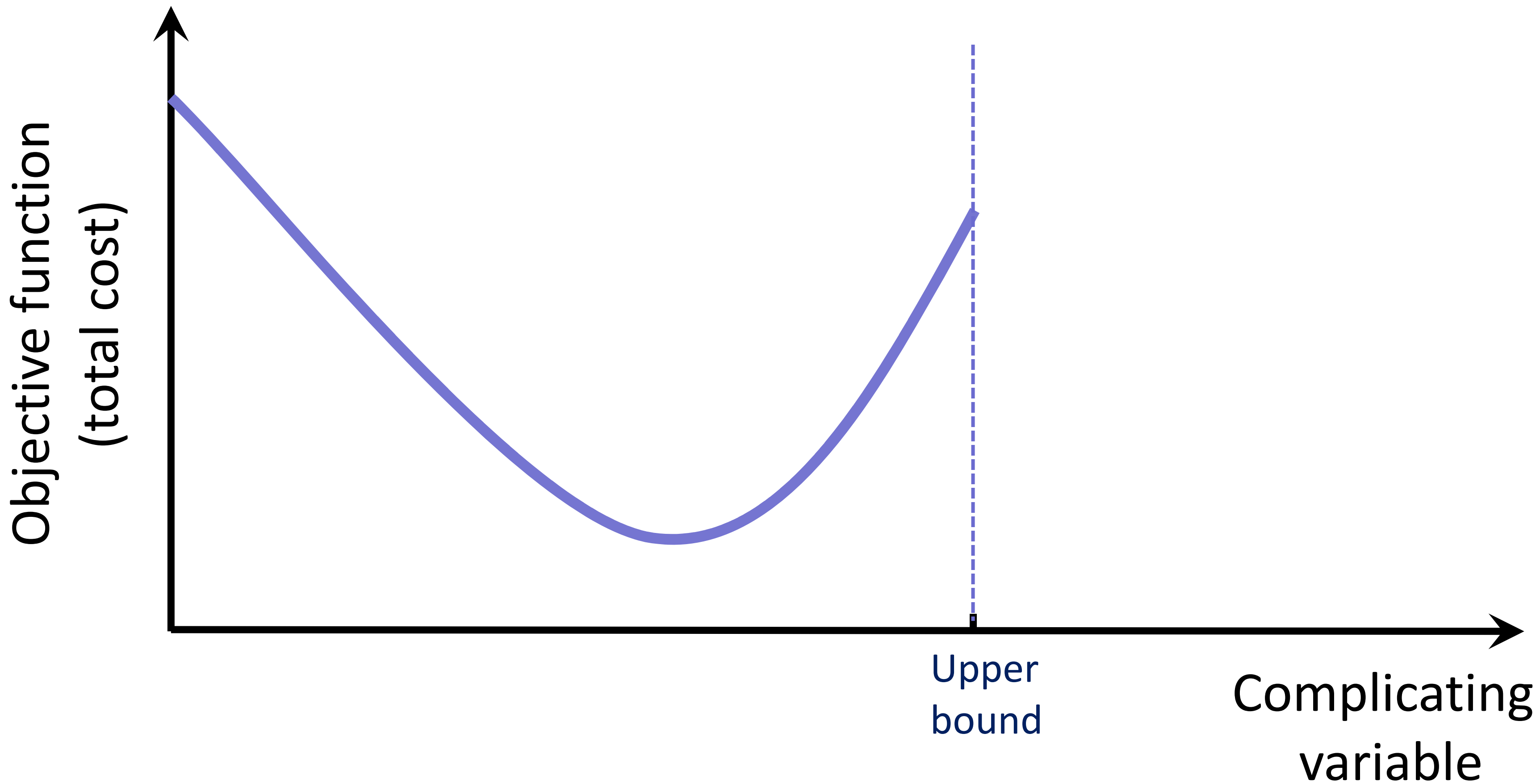
Concept



Potential problems:

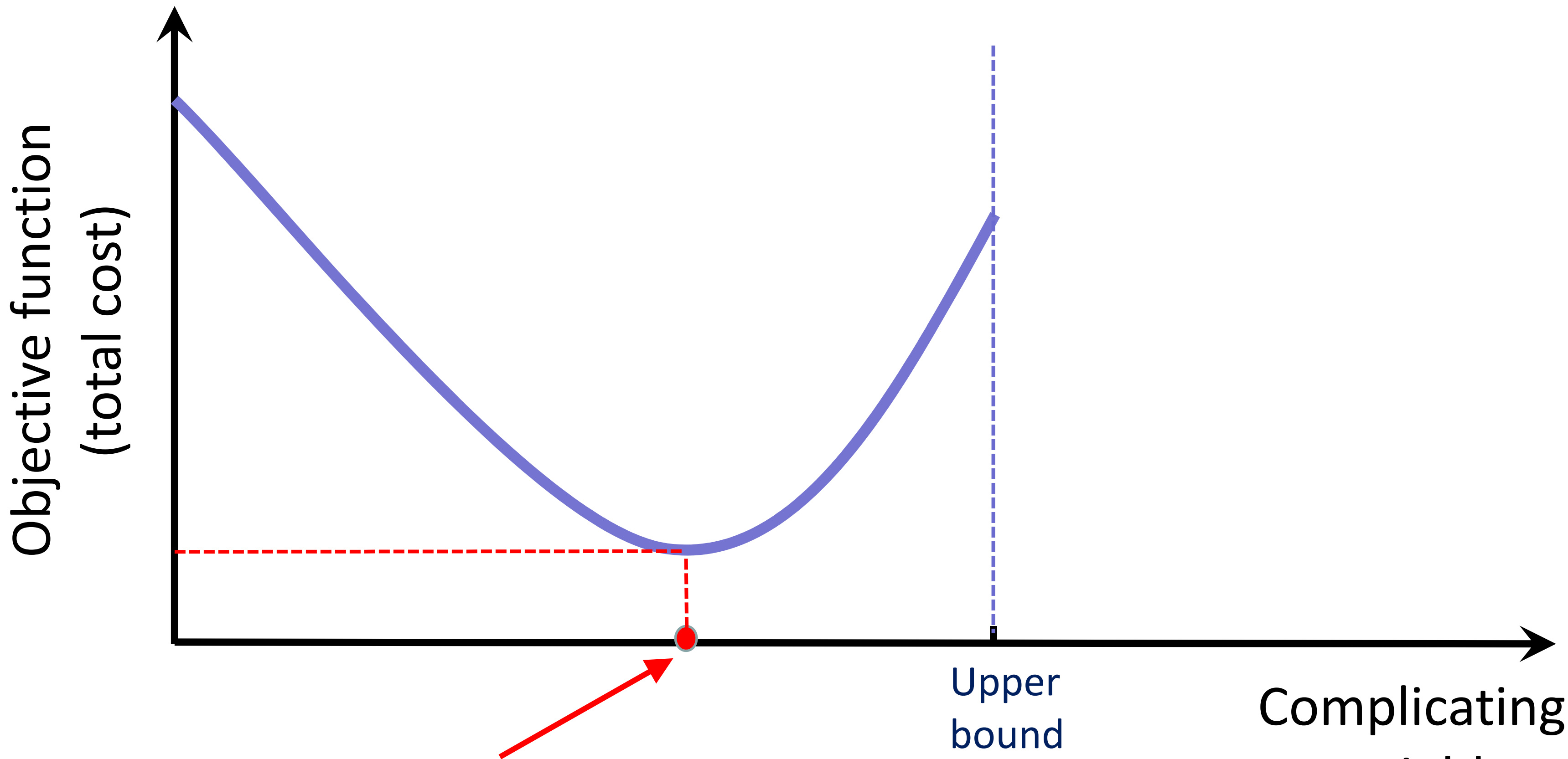
- Investment decisions
- Productions schedules among different markets
- etc

Concept



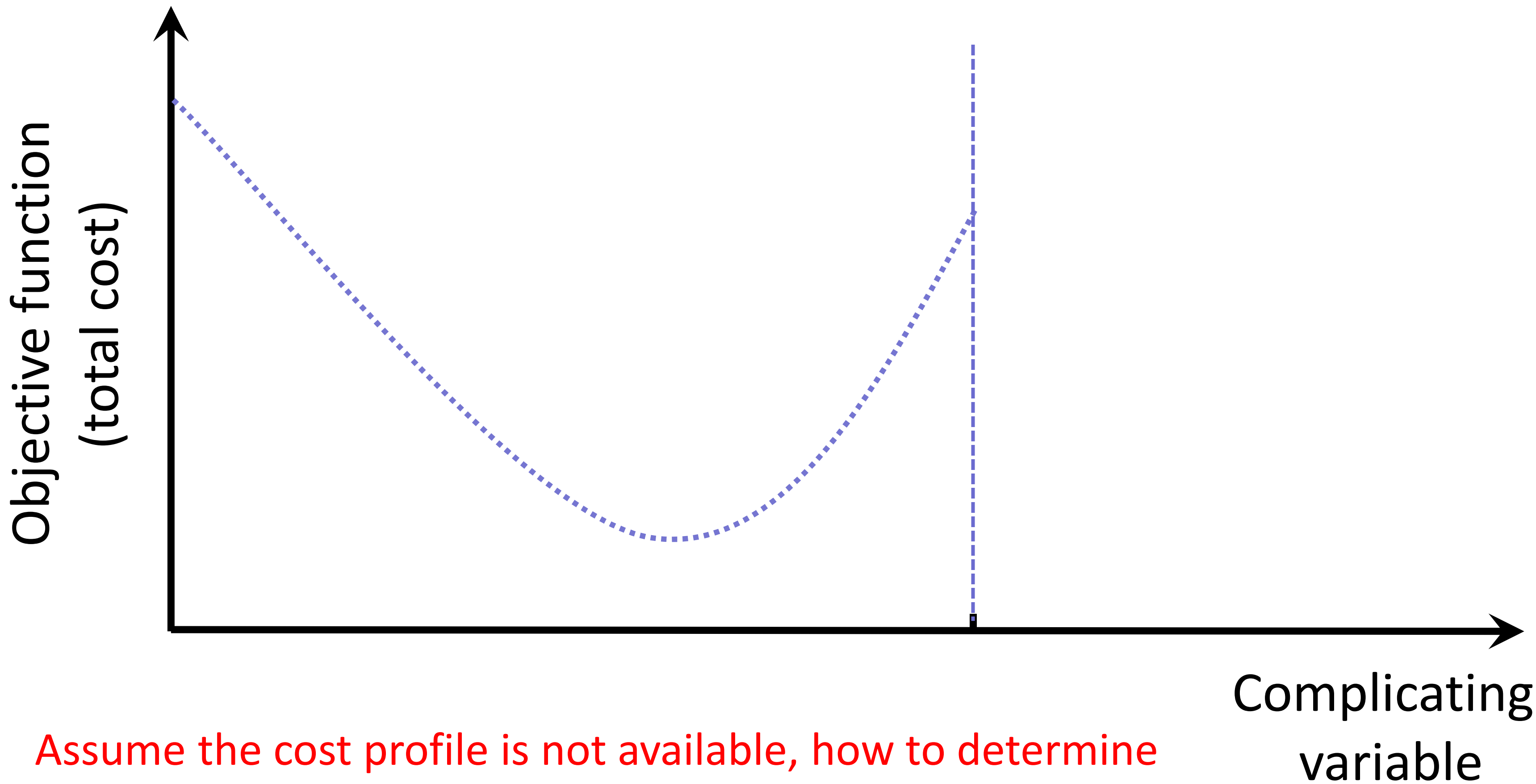
Objective function
(total cost)
Upper
bound
Complicating
variable

Concept



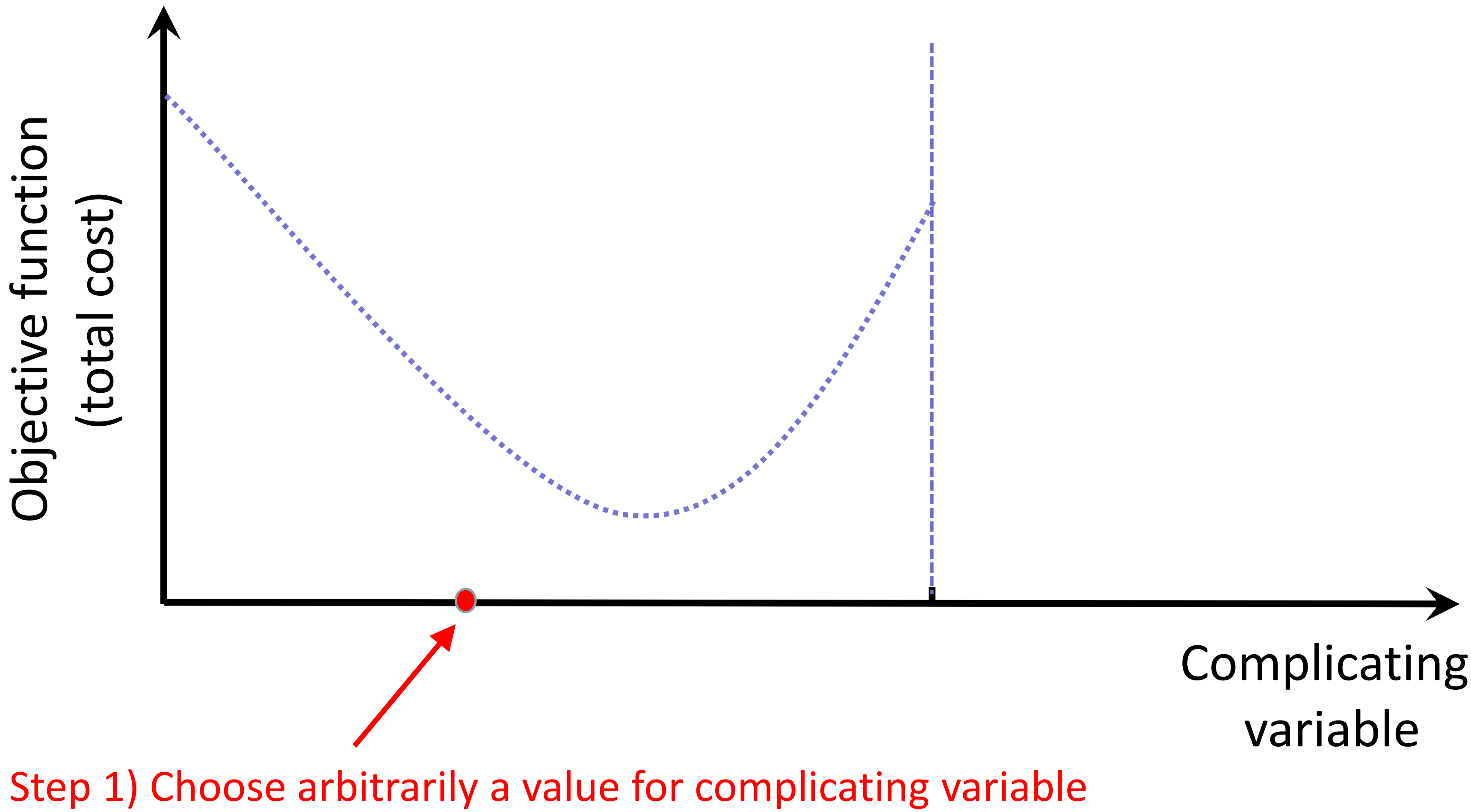
Optimal value for complicating variable
is trivial, if the cost profile is available!
But what if it is not?

Concept



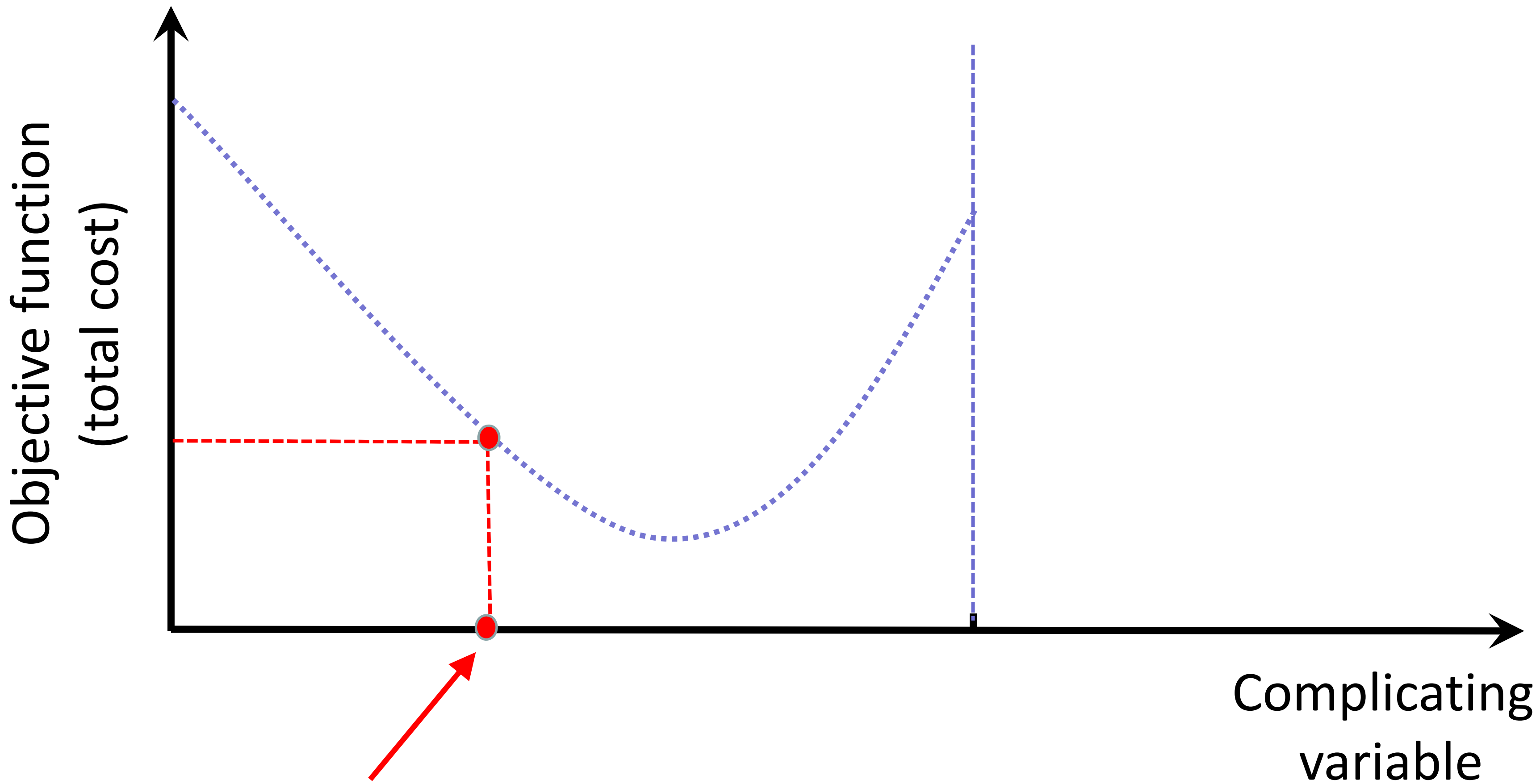
Assume the cost profile is not available, how to determine the optimal point by mathematical tricks?

Concept



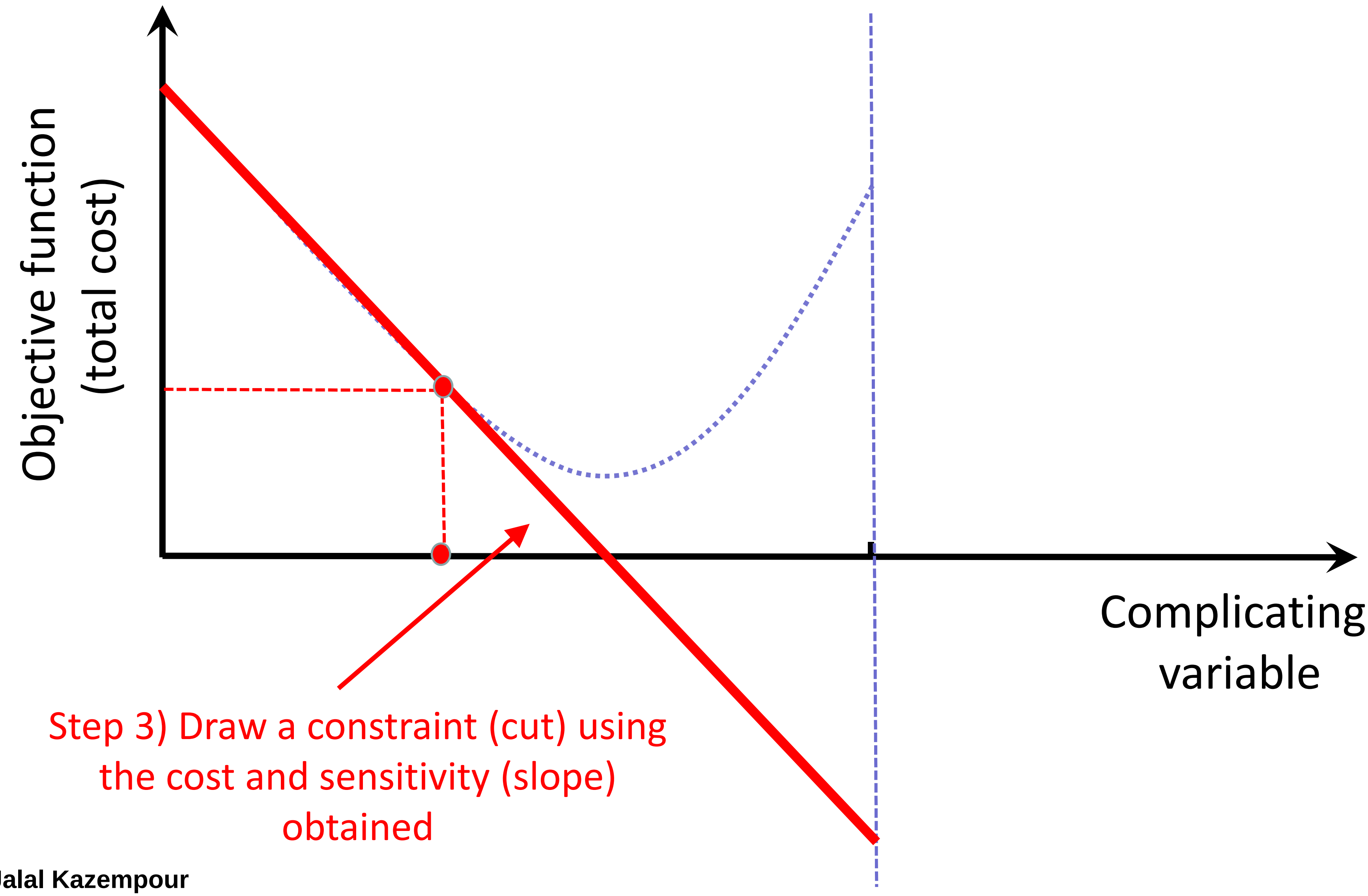
Objective function
(total cost)
Complicating
variable
Step 1) Choose arbitrarily a value for complicating variable

Concept

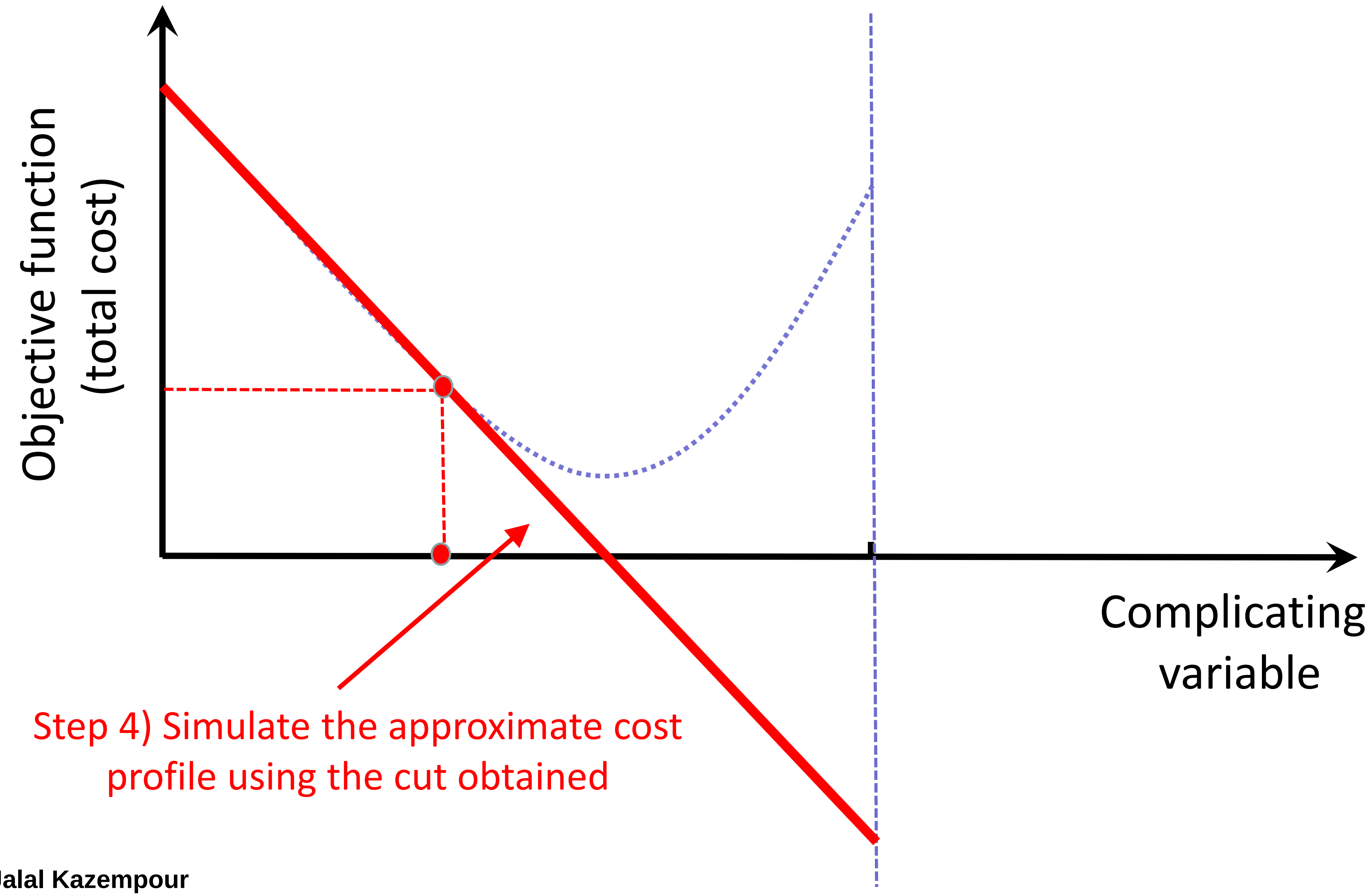


Step 2) Calculate the corresponding i) cost, and ii) sensitivity of cost with respect to selected complicating variable

Concept

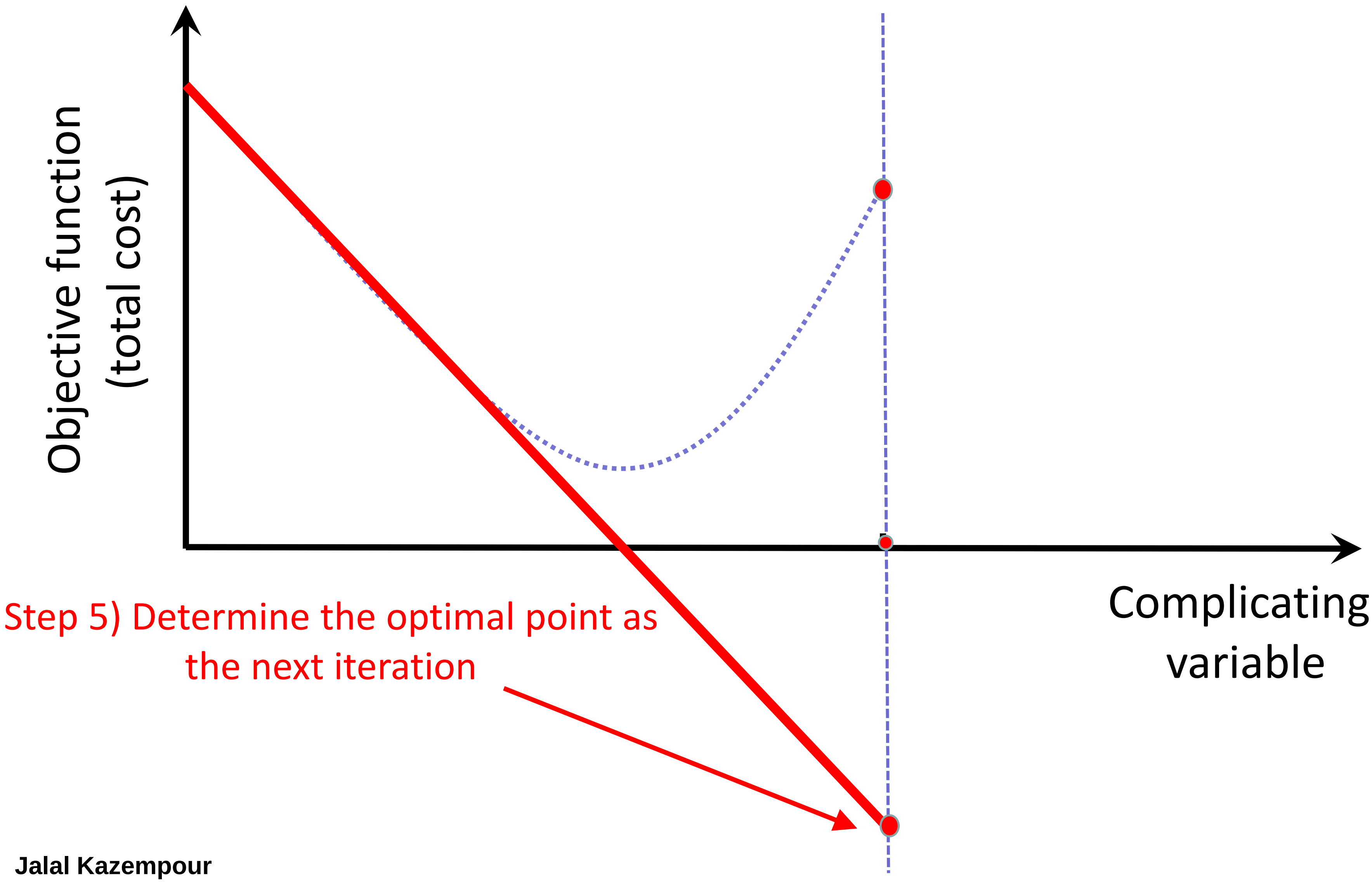

Objective function
(total cost)
Complicating
variable
Step 3) Draw a constraint (cut) using
the cost and sensitivity (slope)
obtained

Concept



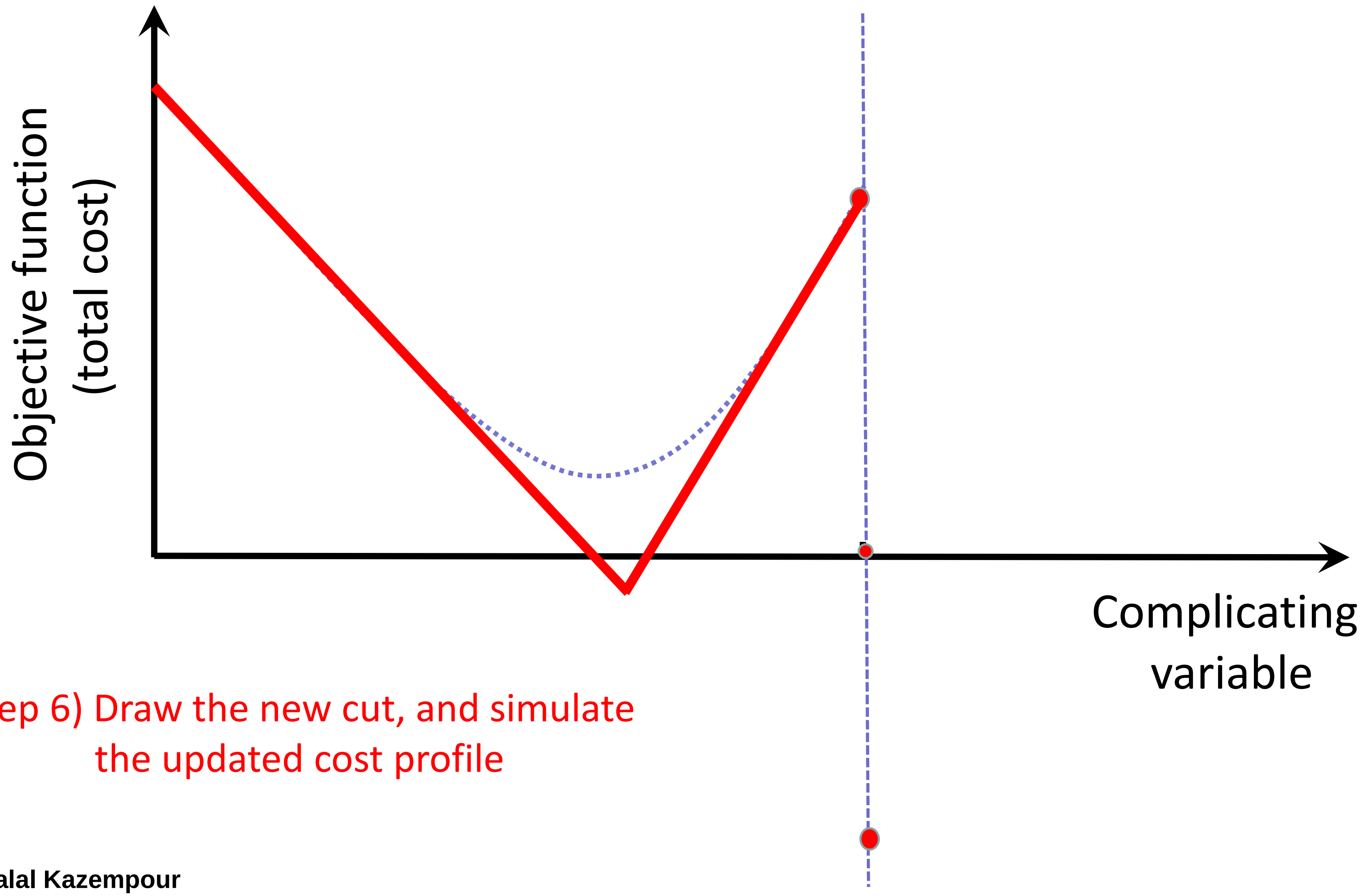
Objective function
(total cost)
Complicating
variable
Step 4) Simulate the approximate cost
profile using the cut obtained

Concept



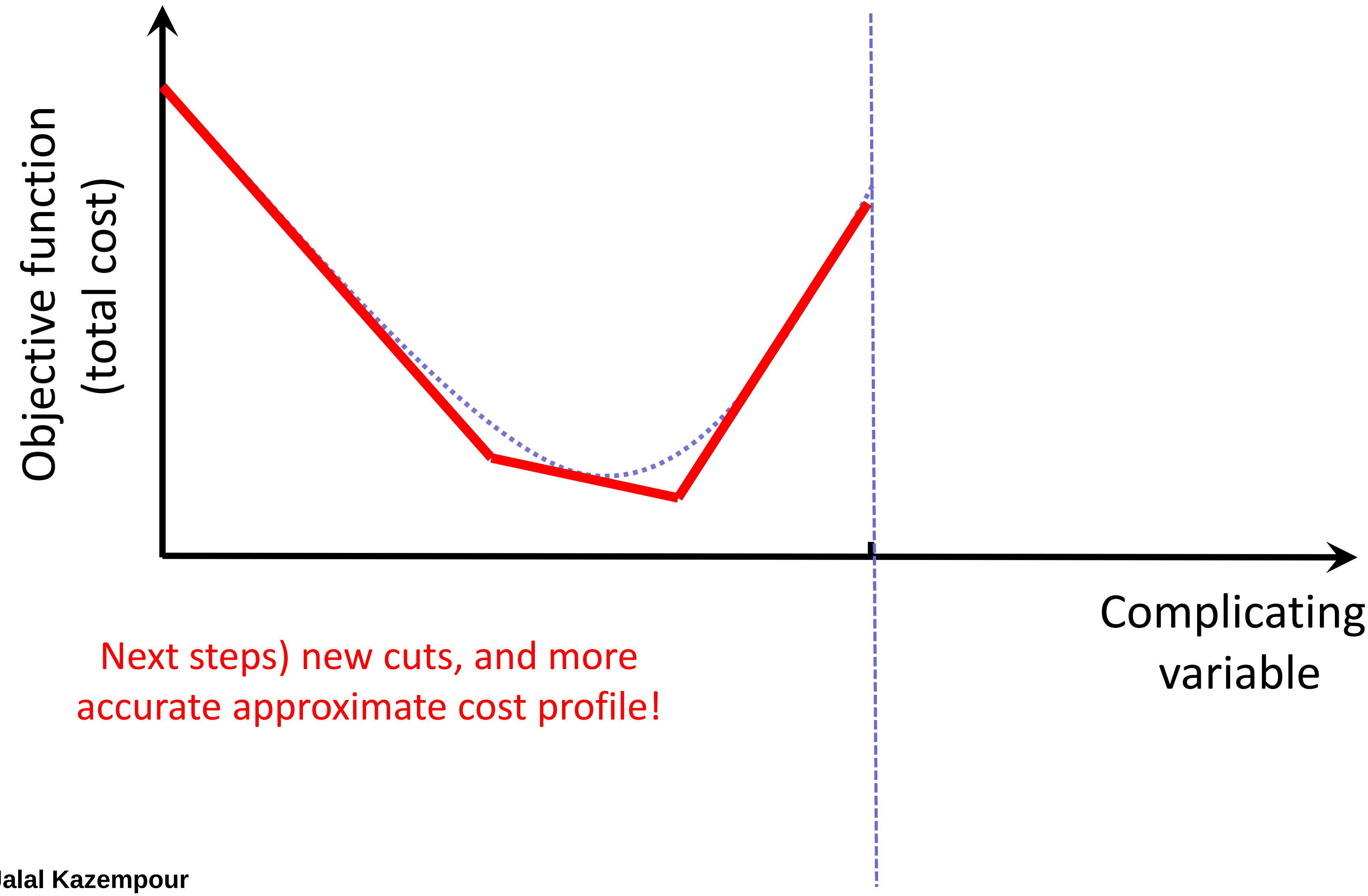
Objective function
(total cost)
Complicating
variable
Step 5) Determine the optimal point as
the next iteration

Concept



Step 6) Draw the new cut, and simulate the updated cost profile

Concept



Next steps) new cuts, and more accurate approximate cost profile!

Two-stage Deterministic Problem

$$\min_{x_1, x_2} \quad c_1 x_1 + c_2\, x_2$$

subject to

$$A_1 x_1 \geq b_1$$

$$E_1 x_1 + A_2 x_2 \geq b_2$$

Two-stage Deterministic Problem

First-stage cost

$$\min_{x_1,x_2} \; c_1 x_1 + c_2\, x_2$$

Second-stage cost

subject to

$$A_1 x_1 \geq b_1$$

← First-stage constraint

$$E_1 x_1 + A_2 x_2 \geq b_2$$

Linking constraint

Two-stage Deterministic Problem

$$\min_{x_1, x_2} \; c_1 x_1 + c_2 x_2$$

subject to

$$A_1 x_1 \geq b_1$$

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First-stage problem:

$$\min_{x_1} \; c_1 x_1 + \alpha_1(x_1)$$

subject to

$$A_1 x_1 \geq b_1$$

Second-stage problem:

$$\alpha_1(x_1) = \min_{x_2} \; c_2 x_2$$

subject to

$$A_2 x_2 \geq b_2 - E_1 x_1$$

$\alpha_1(x_1)$: the second-stage cost as a function of the first-stage decisions x_1 (future cost function)

Two-stage Deterministic Problem

First-stage problem:

$$\min_{x_1} \; c_1 x_1 + \alpha_1(x_1)$$

subject to

$$A_1 x_1 \geq b_1$$

<u>Note</u>: x_1 appears in the second-stage problem!

Second-stage problem:

$$\alpha_1(x_1) = \min_{x_2} \; c_2 x_2$$

subject to

$$A_2 x_2 \geq b_2 - E_1 x_1$$

$\alpha_1(x_1)$: the second-stage cost as a function of the first-stage decisions x_1 (future cost function)

Two-stage Deterministic Problem

One potential solution approach

First-stage problem:

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Two-stage Deterministic Problem

One potential solution approach

- **Step 1) Discretize** x_1 into a set of trial values $\{\hat{x}_{1i}, i = 1, \dots, n\}$
- **Step 2)** Solve the second-stage problem for each of the trial values
- **Step 3)** Construct future cost function $\alpha_1(x_1)$. Intermediate values of $\alpha_1(x_1)$ are obtained by interpolation from the neighboring discretized values.
- **Step 4)** Solve the first-stage problem using the future cost function constructed.

First-stage problem:

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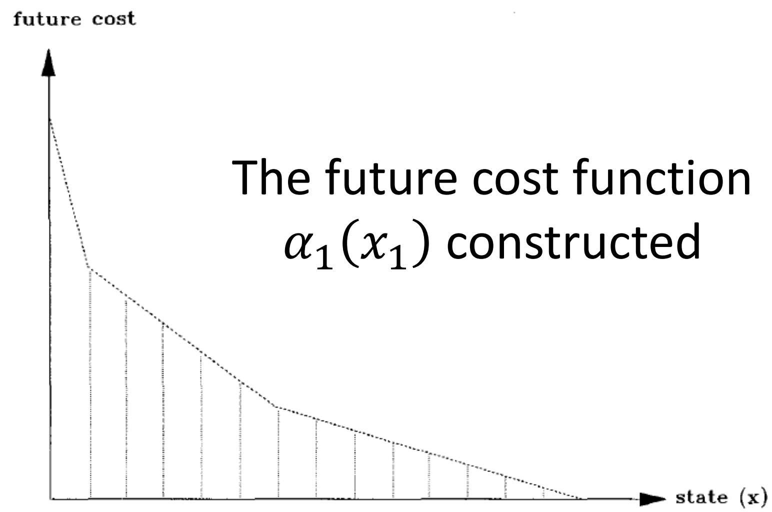

The future cost function $\alpha_1(x_1)$ constructed

Two-stage Deterministic Problem

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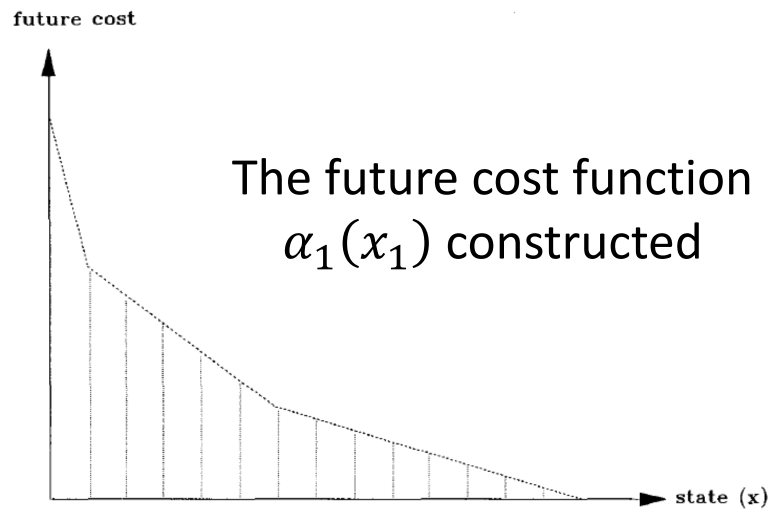

The future cost function $\alpha_1(x_1)$ constructed

What is the name of this technique?

Two-stage Deterministic Problem

One potential solution approach

- **Step 1) Discretize** x_1 into a set of trial values $\{\hat{x}_{1i}, i = 1, \ldots, n\}$
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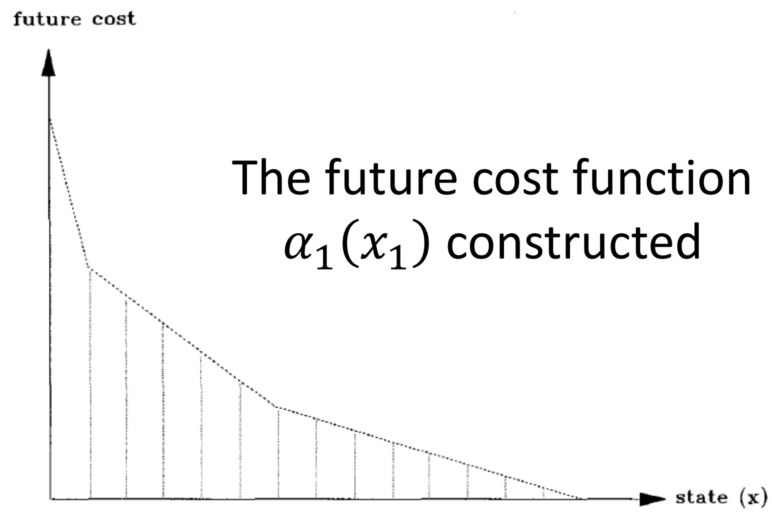


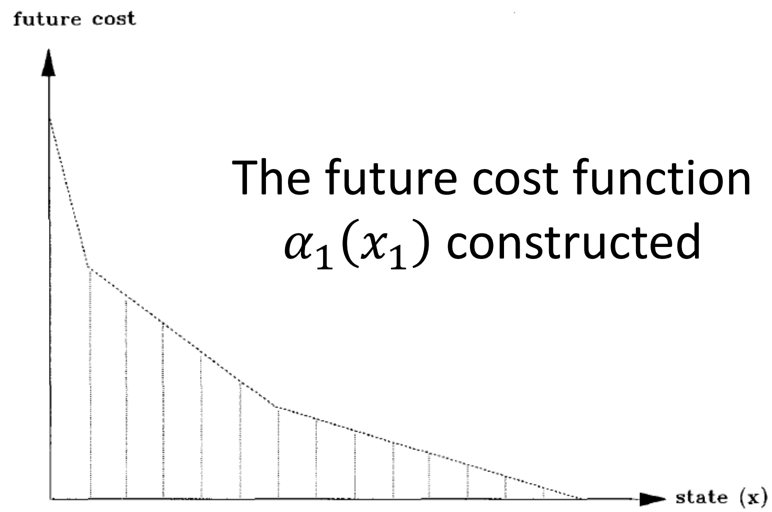
The future cost function $\alpha_1(x_1)$ constructed

What is the name of this technique?

Dynamic programming (DP)

Two-stage Deterministic Problem

One potential solution approach

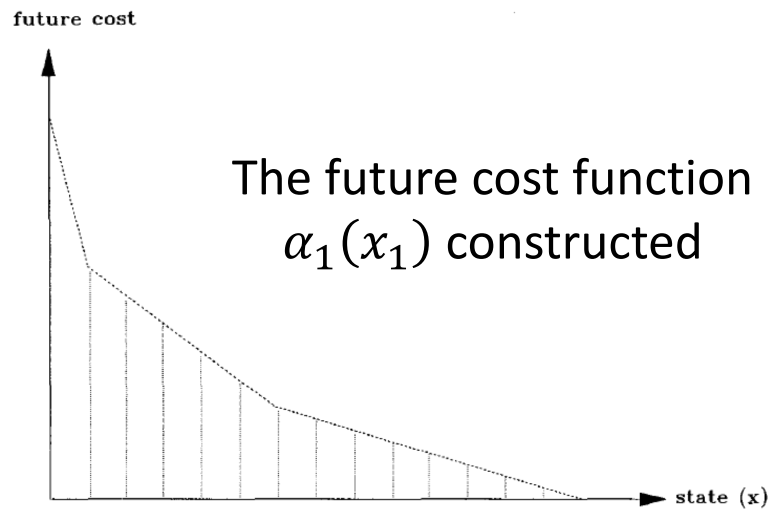


The future cost function $\alpha_1(x_1)$ constructed

What is the main drawback of dynamic programming (DP)?

Two-stage Deterministic Problem

One potential solution approach



The future cost function $\alpha_1(x_1)$ constructed

What is the main drawback of dynamic programming (DP)?

DP needs to discretize the decision variables x_1, which results in computational issues!

For example, 10 decision variables and 4 discretized value for each variable leads to 4^{10} discrete values!

Two-stage Deterministic Problem

Alternative solution approach

Two-stage Deterministic Problem

Alternative solution approach

Dual dynamic programming (DDP) instead of DP!

Advantage:

To approximate the future cost function $\alpha_1(x_1)$ by analytical functions rather than a set of discrete values!

Two-stage Deterministic Problem

DDP functioning procedure

$$\alpha_1(x_1) = \min_{x_2} \; c_2 x_2$$

subject to

$$A_2 x_2 \geq b_2 - E_1 x_1 \quad : \; \pi$$

Two-stage Deterministic Problem

DDP functioning procedure

$$\alpha_1(x_1) = \min_{x_2} \; c_2 x_2$$

$$\text{subject to}$$

$$A_2 x_2 \geq b_2 - E_1 x_1 \quad : \; \pi$$

Dual of the second-stage problem:

$$\max_{\pi} \; \pi(b_2 - E_1 x_1)$$

$$\text{subject to}$$

$$\pi A_2 \geq c_2$$

Two-stage Deterministic Problem

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$$\alpha_1(x_1) = \pi^*(b_2 - E_1 x_1)$$

In the optimal solution (strong duality theorem)

Two-stage Deterministic Problem

DDP functioning procedure

$$\alpha_1(x_1) = \min_{x_2} \; c_2 x_2$$

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In the optimal solution (strong duality theorem)

Interpretation: there is a linear relation between x_1 and the future cost function $\alpha_1(x_1)$ if the sensitivity π^* is known!

Two-stage Deterministic Problem

DDP functioning procedure

$$\alpha_1(x_1) = \pi^*(b_2 - E_1 x_1)$$

Two-stage Deterministic Problem

DDP functioning procedure

$$\alpha_1(x_1) = \pi^*(b_2 - E_1 x_1)$$

Assume π^1, π^2, …, π^n are possible values for π^*. Then, $\alpha_1(x_1)$ can be characterized as follows:

$$\alpha_1(x_1) = \min_{\alpha,\, x_1} \ \alpha$$

$$\text{subject to}$$

$$\alpha \ \geq \ \pi^1(b_2 - E_1 x_1)$$

$$\alpha \ \geq \ \pi^2(b_2 - E_1 x_1)$$

$$\vdots$$

$$\alpha \ \geq \ \pi^n(b_2 - E_1 x_1)$$

Two-stage Deterministic Problem

DDP functioning procedure

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Let's interpret this optimization problem!

Two-stage Deterministic Problem

DDP functioning procedure

$$\alpha_1(x_1) = \pi^*(b_2 - E_1 x_1)$$

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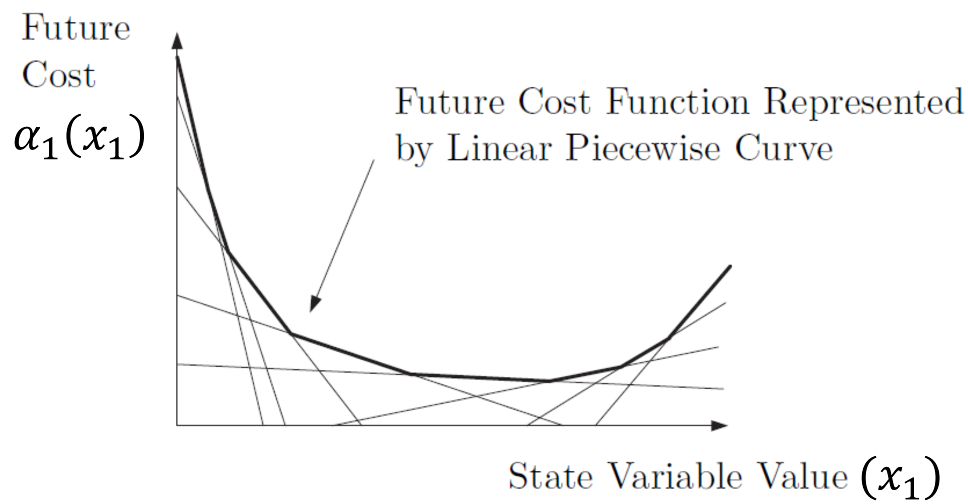

Note: this means that we can construct a piecewise linear function for $\alpha_1(x_1)$ problem (analytically but approximately) without need to discretize x_1!

Two-stage Deterministic Problem

DDP functioning procedure

$$\alpha_1(x_1) = \pi^*(b_2 - E_1 x_1)$$

Assume π^1, π^2, …, π^n are possible values for π^*. Then, $\alpha_1(x_1)$ can be characterized as follows:

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.

.

.

$$\alpha \ \geq \ \pi^n(b_2 - E_1 x_1)$$

Recall the first-stage problem:

$$\min_{x_1} \ c_1 x_1 + \alpha_1(x_1)$$

subject to

$$A_1 x_1 \geq b_1$$

Two-stage Deterministic Problem

DDP functioning procedure

$$\alpha_1(x_1) = \pi^*(b_2 - E_1 x_1)$$

Assume π^1, π^2, …, π^n are possible values for π^*. Then, $\alpha_1(x_1)$ can be characterized as follows:

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subject to

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.

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$$\alpha \geq \pi^n(b_2 - E_1 x_1)$$

Recall the first-stage problem:

$$\min_{x_1} \ c_1 x_1 + \alpha_1(x_1)$$

subject to

$$A_1 x_1 \geq b_1$$

Let's merge them!

Two-stage Deterministic Problem

DDP functioning procedure

Updated first-stage problem including the piecewise linear function $\boldsymbol{\alpha_1(x_1)}$

$$
\begin{aligned}
&\min_{\alpha,\, x_1} \;\; c_1 x_1 + \alpha \\
&\text{subject to} \\
&A_1 x_1 \geq b_1 \\
&\alpha \;\geq\; \pi^1 (b_2 - E_1 x_1) \\
&\alpha \;\geq\; \pi^2 (b_2 - E_1 x_1) \\
&\qquad\quad \vdots \\
&\alpha \;\geq\; \pi^n (b_2 - E_1 x_1)
\end{aligned}
$$

Two-stage Deterministic Problem

DDP functioning procedure

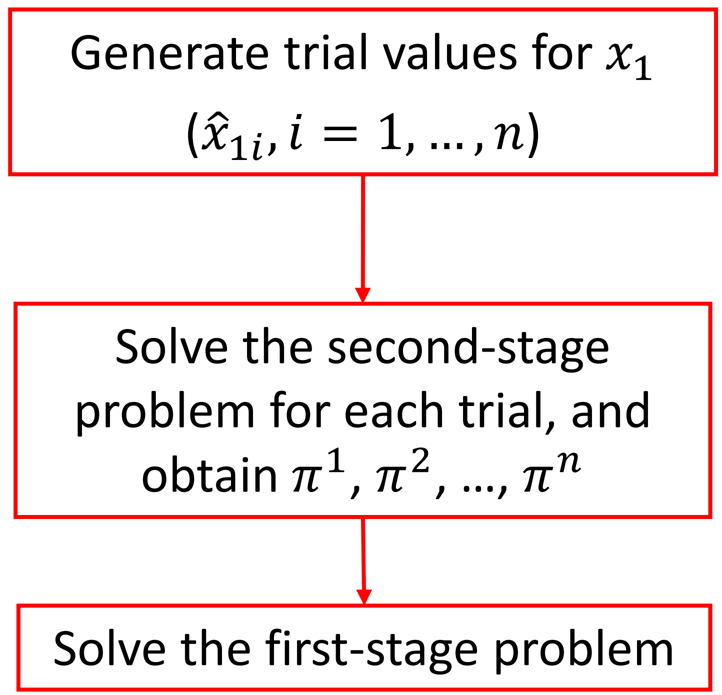
How to generate possible values for π^*, i.e., $\pi^1, \pi^2, \ldots, \pi^n$?

Two-stage Deterministic Problem

DDP functioning procedure

How to generate possible values for π^*, i.e., $\pi^1, \pi^2, \ldots, \pi^n$?

Option 1:



Two-stage Deterministic Problem

DDP functioning procedure

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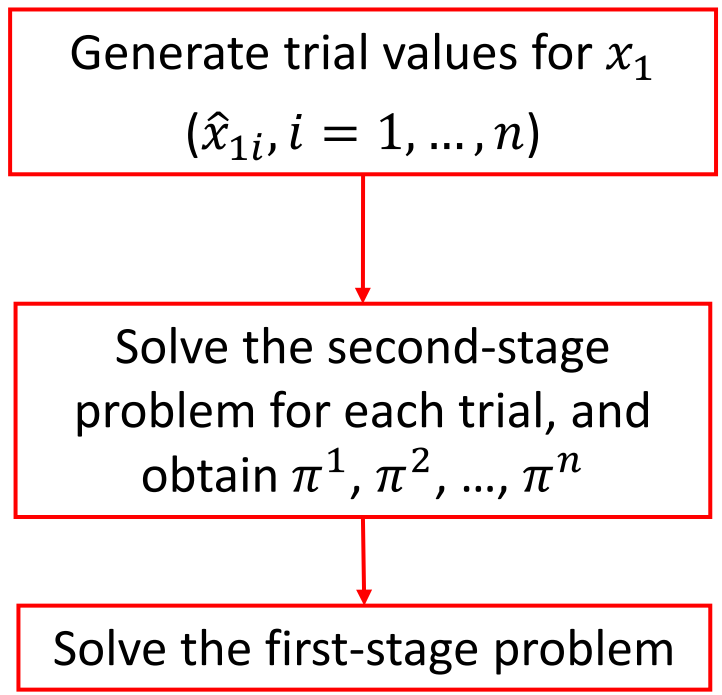


Do you recommend this option?

Two-stage Deterministic Problem

DDP functioning procedure

How to generate possible values for π^*, i.e., π^1, π^2, …, π^n ?

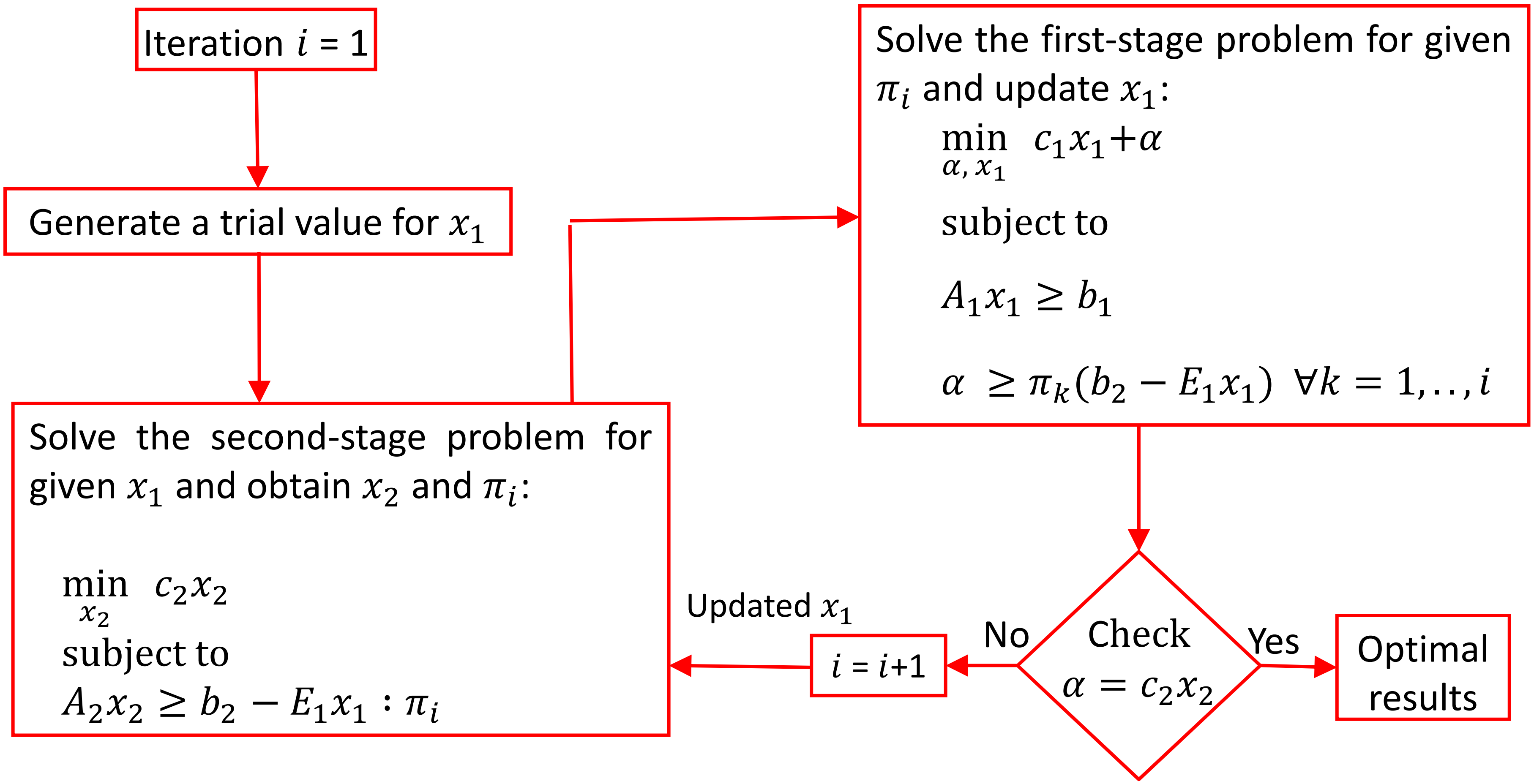
Option 2 (systematic iterative approach):

Two-stage Deterministic Problem

DDP functioning procedure

How to generate possible values for π^*, i.e., $\pi^1, \pi^2, \ldots, \pi^n$?

Option 2 (systematic iterative approach):



Two-stage Deterministic Problem

DDP functioning procedure

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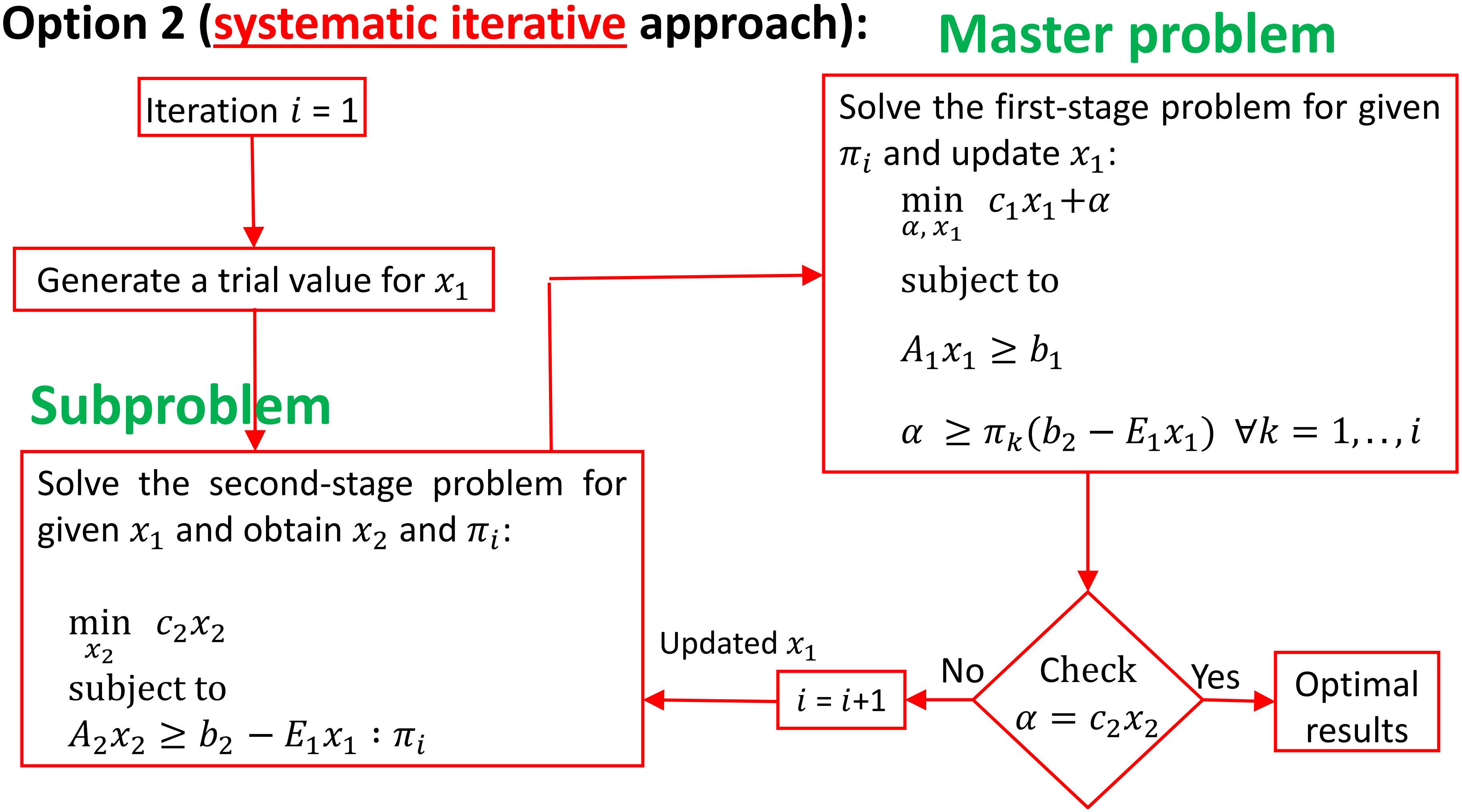

How to generate possible values for π^*, i.e., $\pi^1, \pi^2, \ldots, \pi^n$?

Option 2 (systematic iterative approach):

This approach is indeed Benders' decomposition!

Important note:

We can guarantee obtaining the global optimal solution by Benders' decomposition, **if** the objective function of the original (non-decomposed) problem is **convex** with respect to the complicating variable!

Two-stage Deterministic Problem

Simple Example

$$\underset{x,y}{\text{minimize}} \quad z = -y - x/4$$

$$\begin{aligned}
y \quad -x &\leq 5 \\
y \quad -\tfrac{1}{2}x &\leq \tfrac{15}{2} \\
y \quad +\tfrac{1}{2}x &\leq \tfrac{35}{2} \\
-y \quad +x &\leq 10 \\
0 &\leq x \leq 16 \\
y &\geq 0\,.
\end{aligned}$$

Two-stage Deterministic Problem

Simple Example

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0 &\leq x \leq 16 \\
y &\geq 0\,.
\end{aligned}$$

Let's consider *x* as the complicating variable!

Two-stage Deterministic Problem

Simple Example

i : current Benders' iteration

k: set of previous iterations

Subproblem:

$$\underset{y^{(i)}}{\text{minimize}} \; -y^{(i)}$$

subject to

$$y^{(i)} \leq 5 + x^{\text{fixed }(i)} \quad : \pi^{(i)}$$

$$y^{(i)} \leq \frac{15}{2} + \frac{x^{\text{fixed }(i)}}{2} \quad : \mu^{(i)}$$

$$y^{(i)} \leq \frac{35}{2} - \frac{x^{\text{fixed }(i)}}{2} \quad : \sigma^{(i)}$$

$$y^{(i)} \leq 10 - x^{\text{fixed }(i)} \quad : \gamma^{(i)}$$

$$y^{(i)} \geq 0$$

Master problem:

$$\underset{x^{(i)},\, \alpha^{(i)}}{\text{minimize}} \; -\frac{x^{(i)}}{4} + \alpha^{(i)}$$

subject to

$$0 \leq x^{(i)} \leq 16$$

$$\alpha^{(i)} \geq \alpha^{\text{down}}$$

$$\alpha^{(i)} \geq \pi^{(k)}\left[5 + x^{(i)}\right] + \mu^{(k)}\left[\frac{15}{2} + \frac{x^{(i)}}{2}\right] + \sigma^{(k)}\left[\frac{35}{2} - \frac{x^{(i)}}{2}\right]$$

$$+ \gamma^{(k)}\left[10 - x^{(i)}\right] \quad \forall k = 1, .., i-1$$

Two-stage Deterministic Problem

Simple Example

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$$\underset{y^{(i)}}{\text{minimize}} \; -y^{(i)}$$

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$$y^{(i)} \leq 10 - x^{\text{fixed}\,(i)} \quad : \gamma^{(i)}$$

$$y^{(i)} \geq 0$$

i : current Benders' iteration

k: set of previous iterations

Note: In subproblem, symbols following colon are dual variables (sensitivities).

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Two-stage Deterministic Problem

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Note: In subproblem, symbols following colon are dual variables (sensitivities).

Note: The last constraint of master problem generate "cuts".

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$$0 \leq x^{(i)} \leq 16$$

$$\alpha^{(i)} \geq \alpha^{\text{down}}$$

$$\alpha^{(i)} \geq \pi^{(k)}\left[5 + x^{(i)}\right] + \mu^{(k)}\left[\frac{15}{2} + \frac{x^{(i)}}{2}\right] + \sigma^{(k)}\left[\frac{35}{2} - \frac{x^{(i)}}{2}\right] + \gamma^{(k)}\left[10 - x^{(i)}\right] \quad \forall k = 1, .., i-1$$

Two-stage Deterministic Problem

Simple Example

Algorithm:

- **Step 0: Initialization**

  Set $i = 1$, $x^{\text{fixed (1)}} = x^{\text{initial}}$, and lower bound (LB) = $-\infty$

- **Step 1: Solve subproblem(s):** obtain the values of all dual variables (sensitivities), and the value of objective function, which is upper bound (UB)

- **Step 2: Convergence check**

  If $|\text{UB} - \text{LB}| \leq \epsilon$, then the optimal solution with a level of accuracy ϵ is obtained, otherwise $i \leftarrow i + 1$

- **Step 3: Solve master problem:** obtain the updated $x^{(i)}$ and the value of $\alpha^{(i)}$ as the updated LB, and go Step 1 with the updated fixed x

A more compact form of Benders' decomposition:

Subproblem:

$$\underset{x^{(i)},\, y^{(i)}}{\text{minimize}} \; -y^{(i)}$$

subject to

$$y^{(i)} - x^{(i)} \leq 5$$

$$y^{(i)} - \frac{x^{(i)}}{2} \leq \frac{15}{2}$$

$$y^{(i)} + \frac{x^{(i)}}{2} \leq \frac{35}{2}$$

$$y^{(i)} + x^{(i)} \leq 10$$

$$y^{(i)} \geq 0$$

$$x^{(i)} = x^{\text{fixed }(i)} \;\; : \rho^{(i)}$$

Master problem:

$$\underset{x^{(i)},\, \alpha^{(i)}}{\text{minimize}} \; -\frac{x^{(i)}}{4} + \alpha^{(i)}$$

subject to

$$0 \leq x^{(i)} \leq 16$$

$$\alpha^{(i)} \geq \alpha^{\text{down}}$$

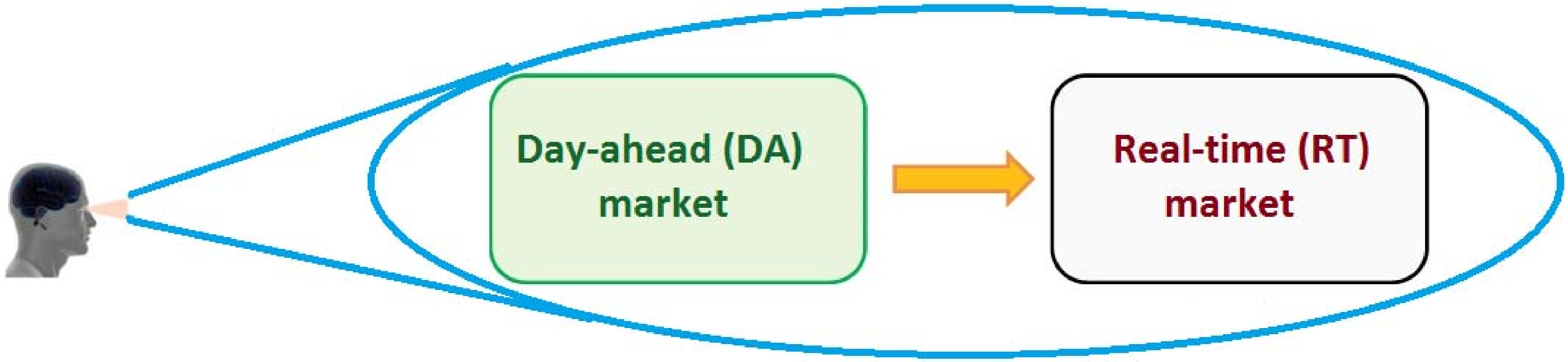
$$\alpha^{(i)} \geq \text{-}y^{(k)} + \rho^{(k)}\left[x^{(i)} - x^{(k)}\right] \quad \forall k = 1, \ldots, i-1$$

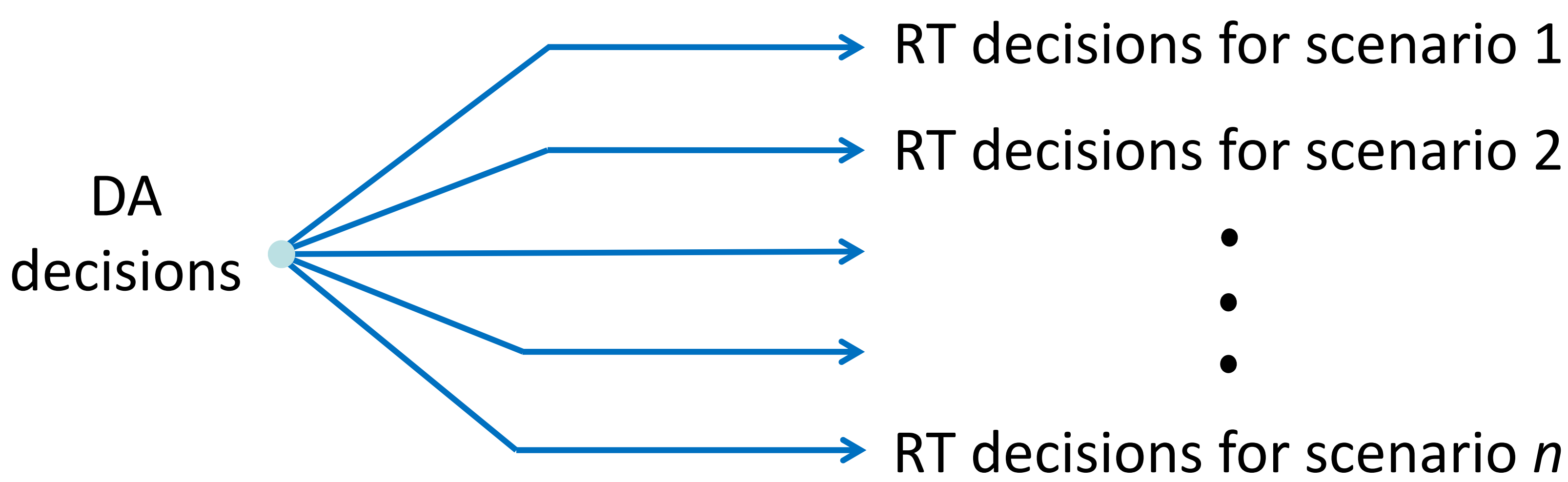
Two-stage Stochastic Problem

Two-stage (day-ahead and real-time) stochastic OPF problem

Two-stage Stochastic Problem

Two-stage (day-ahead and real-time) stochastic OPF problem





Two-stage Stochastic Problem

Two-stage (day-ahead and real-time) stochastic OPF problem

Day-ahead (DA) decisions:

Real-time (RT) decisions:

Two-stage Stochastic Problem

Two-stage (day-ahead and real-time) stochastic OPF problem

Day-ahead (DA) decisions:

- Power schedule [MW] of each generator ($\forall g: 1, .., G$), which is P_g.

Note: this variable is **scenario-independent**, in the sense that it should be adapted to all foreseen scenarios (here-and-now decisions).

Real-time (RT) decisions:

Two-stage Stochastic Problem

Two-stage (day-ahead and real-time) stochastic OPF problem

Day-ahead (DA) decisions:

- Power schedule [MW] of each generator ($\forall g: 1, .., G$), which is P_g.

Note: this variable is **scenario-independent**, in the sense that it should be adapted to all foreseen scenarios (here-and-now decisions).

Real-time (RT) decisions:

(For given DA decisions)

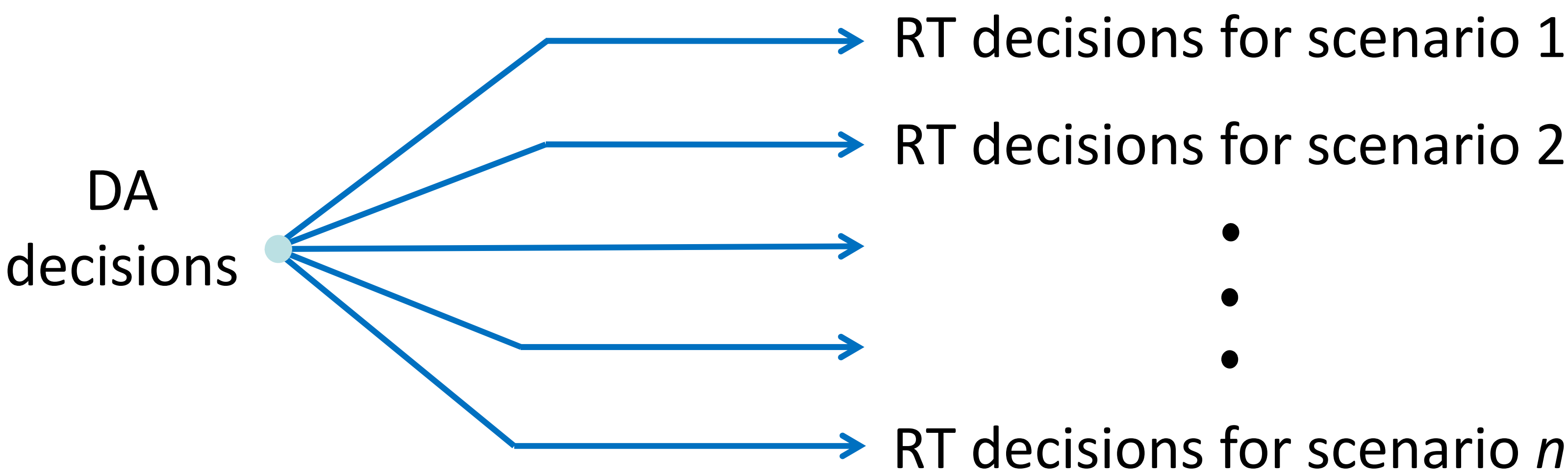
- Reserve deployment [MW] of each generator g under each foreseen scenario ($\forall s: 1, .., S$), which is $r_{g,s}$.
- Load shedding, wind curtailment, etc (all indexed by s).

Note: this variable is **scenario-dependent** (wait-and-see decisions).

Two-stage Stochastic Problem

Two-stage (day-ahead and real-time) stochastic OPF problem

- Is there any <u>complicating variable</u> in this two-stage stochastic problem?



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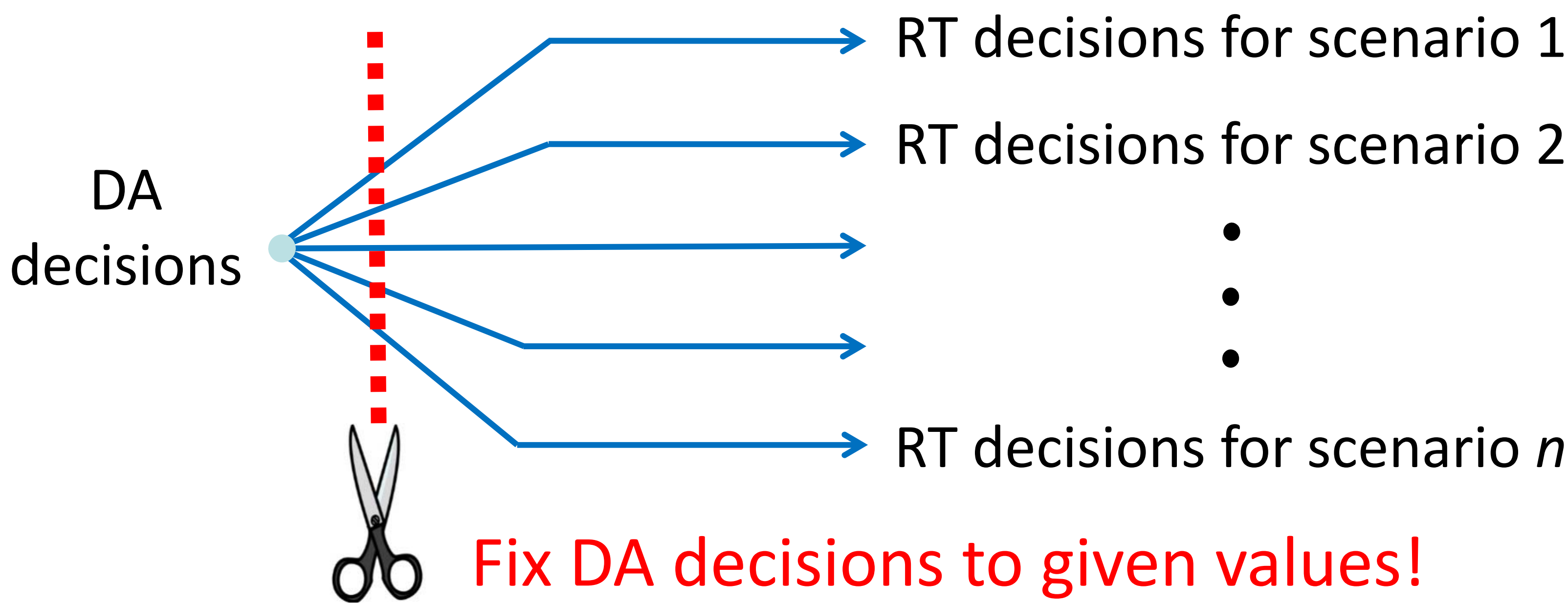

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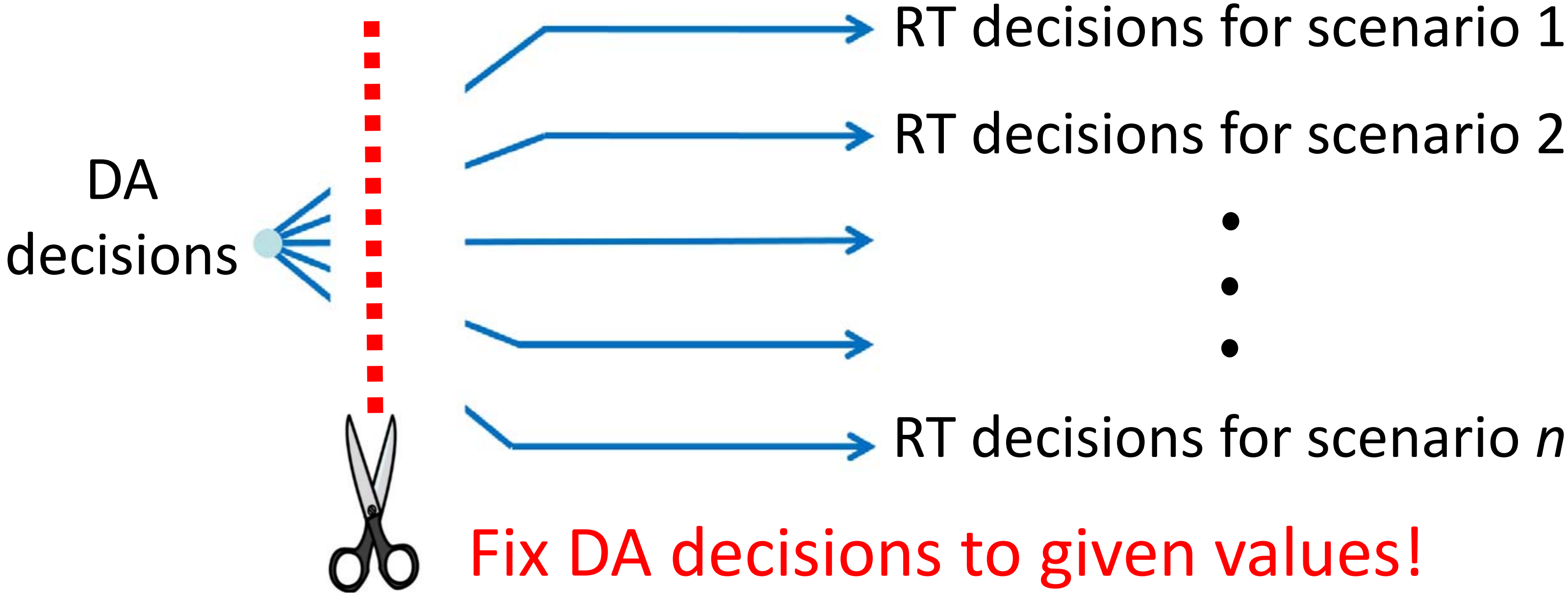

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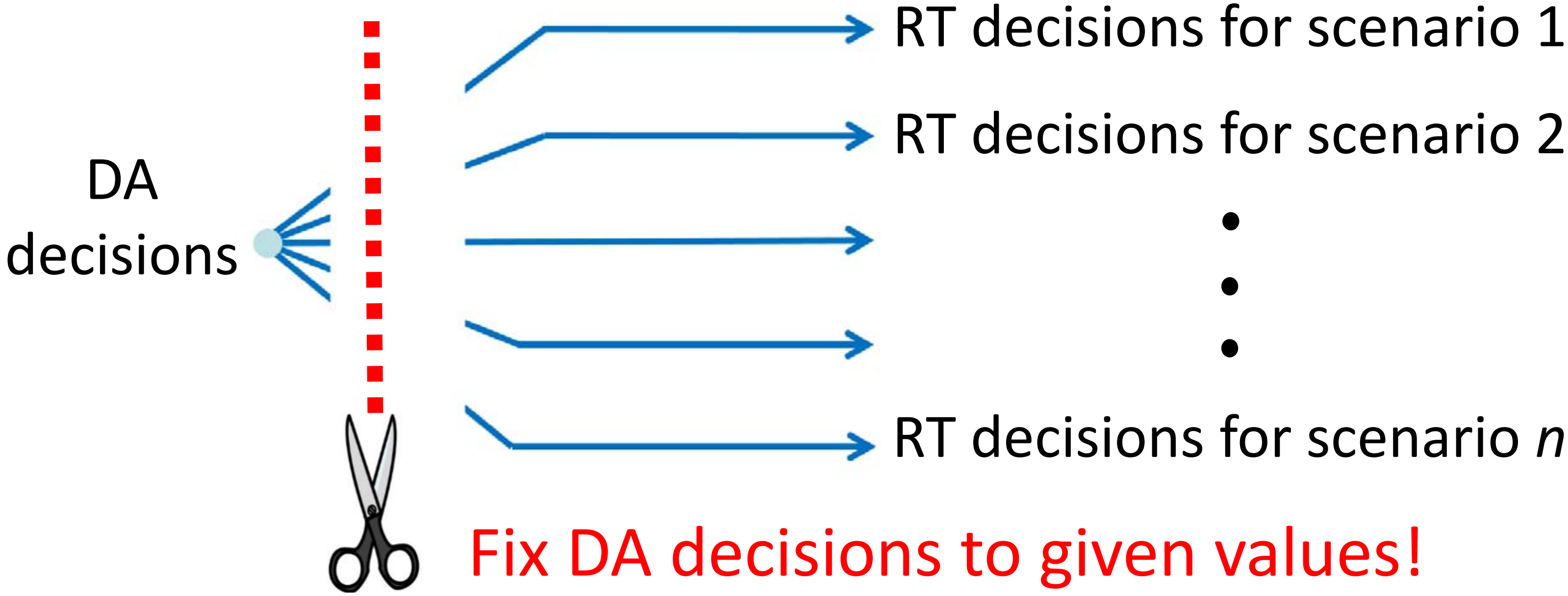


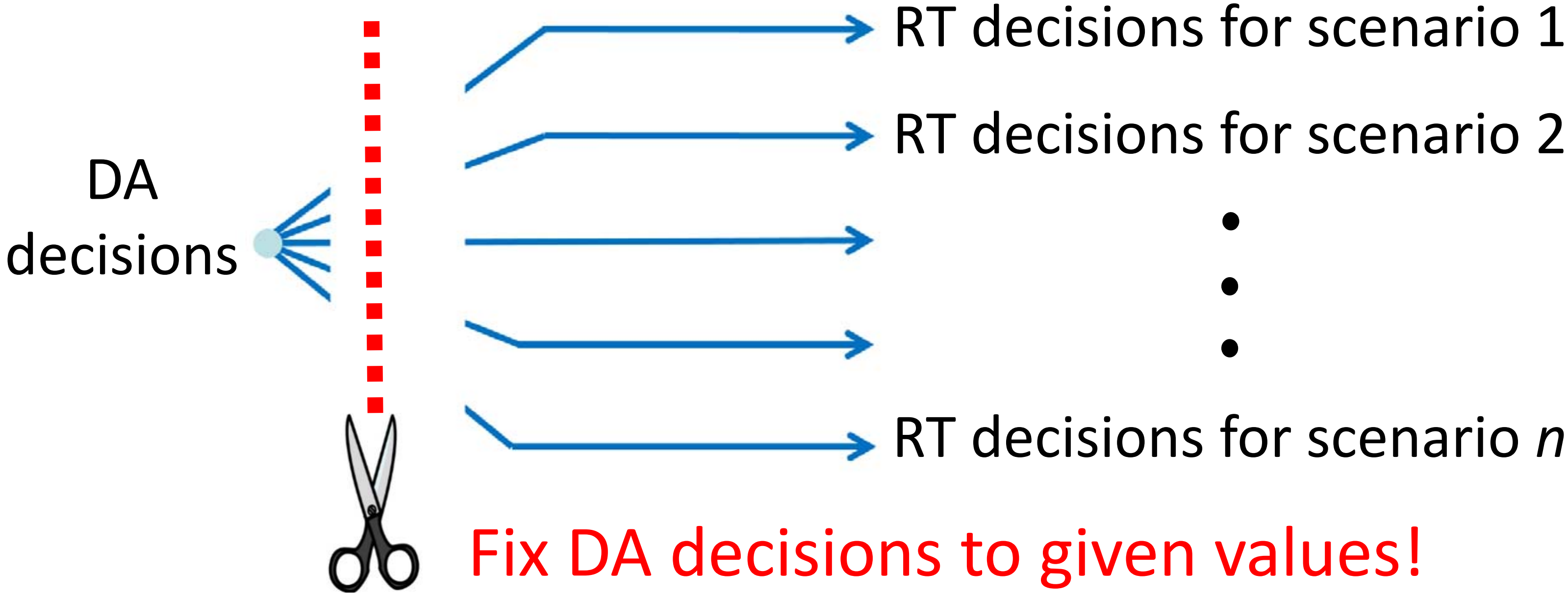
Fix DA decisions to given values!

Then, how many subproblems will you have?

Two-stage Stochastic Problem

Two-stage (day-ahead and real-time) stochastic OPF problem

- Is there any <u>complicating variable</u> in this two-stage stochastic problem?

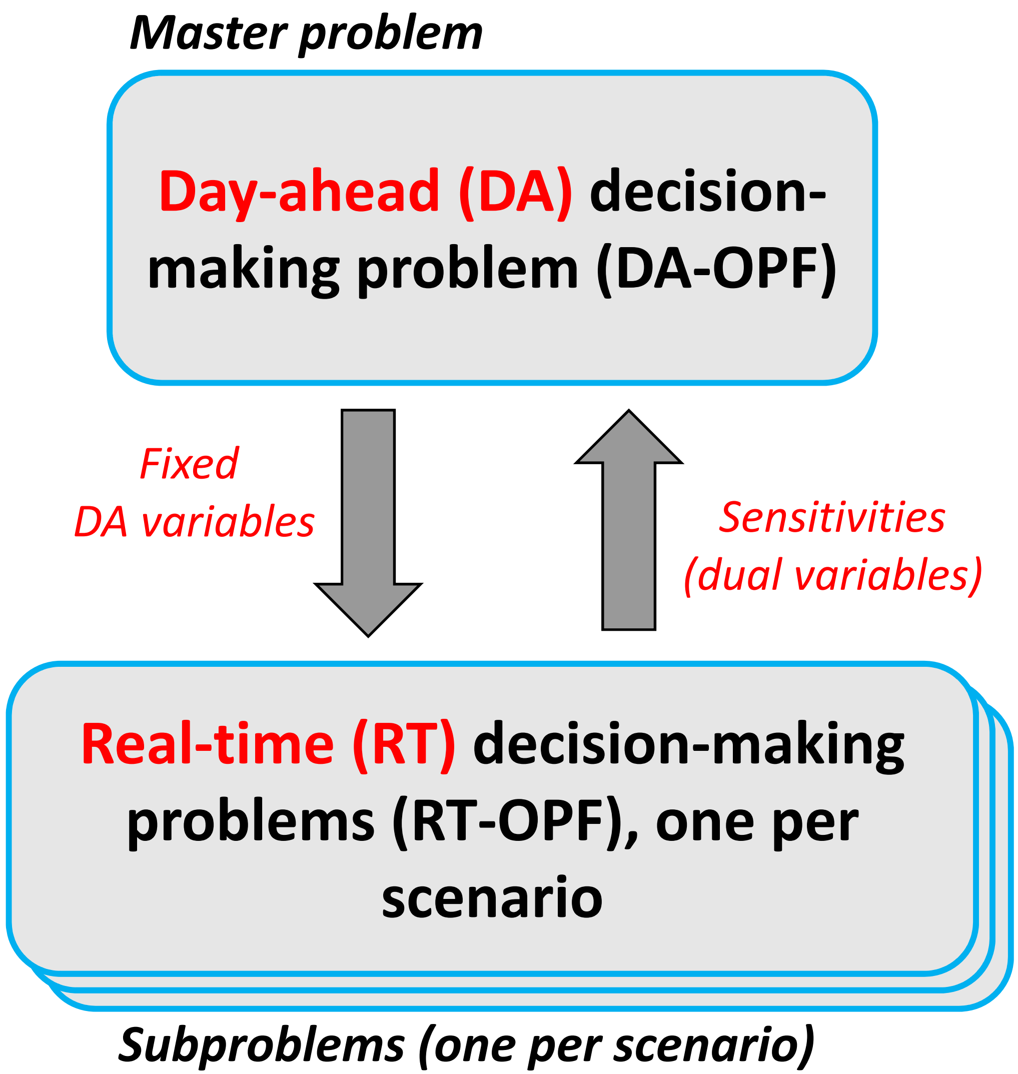


Fix DA decisions to given values!

Then, how many subproblems will you have? **One per scenario!**

Two-stage Stochastic Problem

Two-stage (day-ahead and real-time) stochastic OPF problem



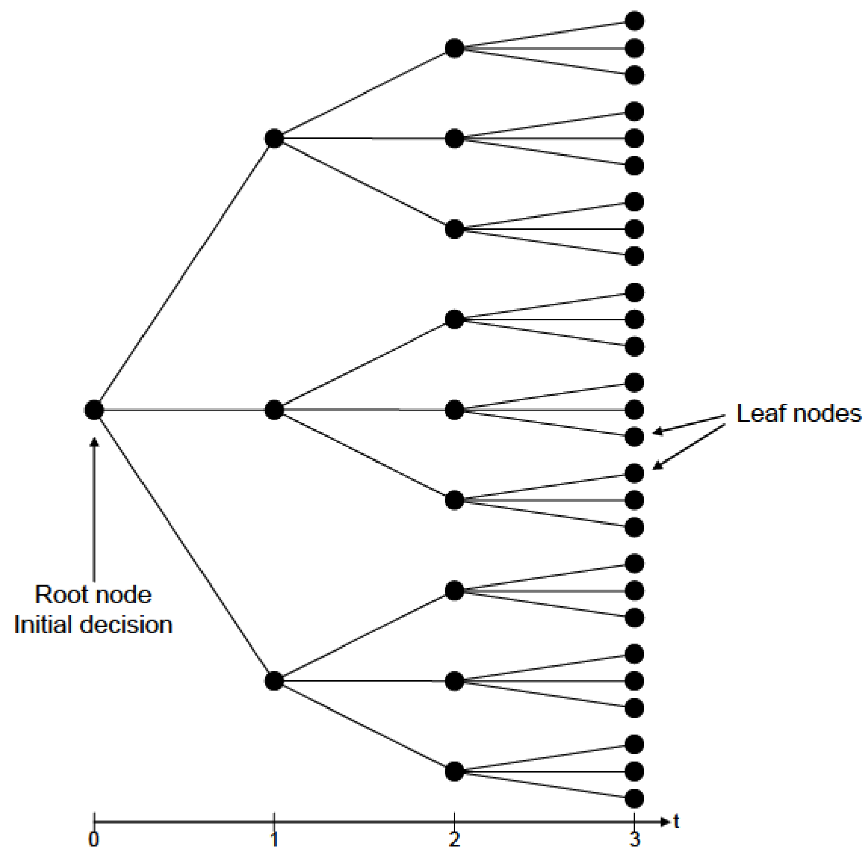
Two-stage Stochastic Problem

Two-stage (day-ahead and real-time) stochastic OPF problem

The application of Benders' decomposition to two-stage stochastic problems is also referred in the literature as **L-shaped decomposition**!

Some more thoughts

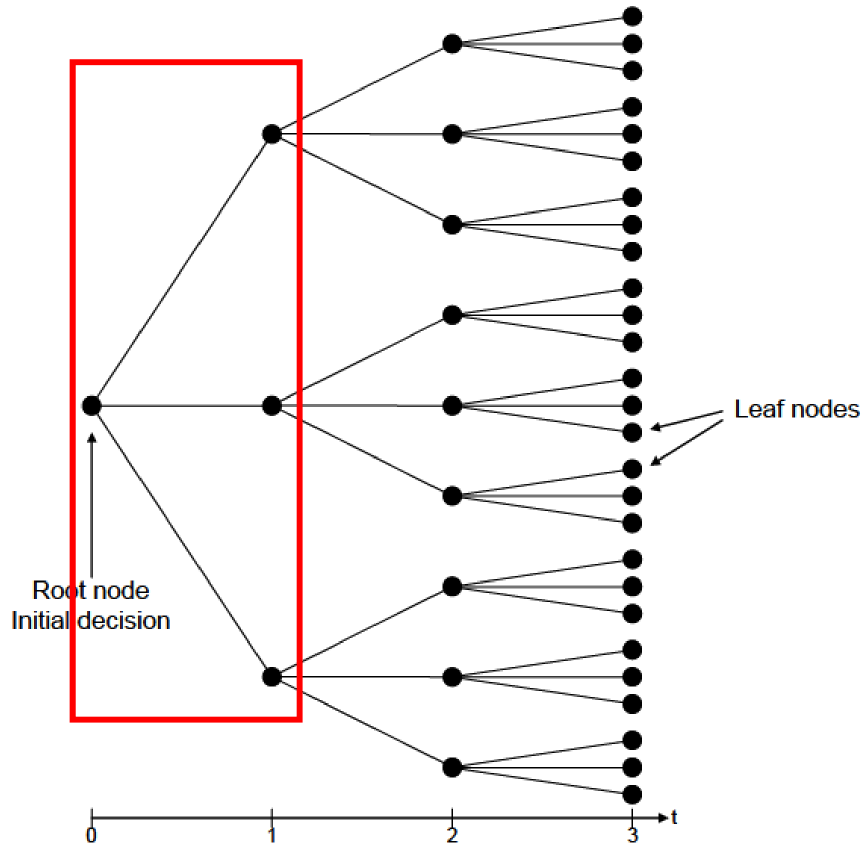
What to do in case of multi-stage (e.g., 4-stage) stochastic problem?



Some more thoughts

What to do in case of multi-stage (e.g., 4-stage) stochastic problem?

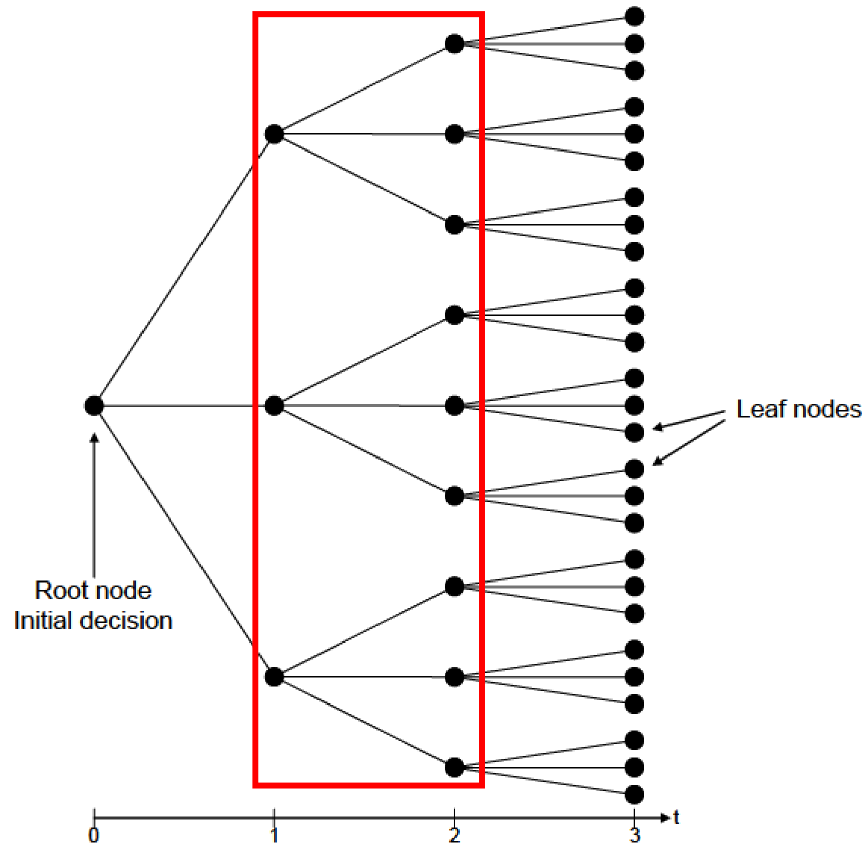
- **Step 1 in each iteration:** Solve a single two-stage stochastic problem
- **Master problem**: Stage 1
- **Subproblems:** Stage 2



Some more thoughts

What to do in case of multi-stage (e.g., 4-stage) stochastic problem?

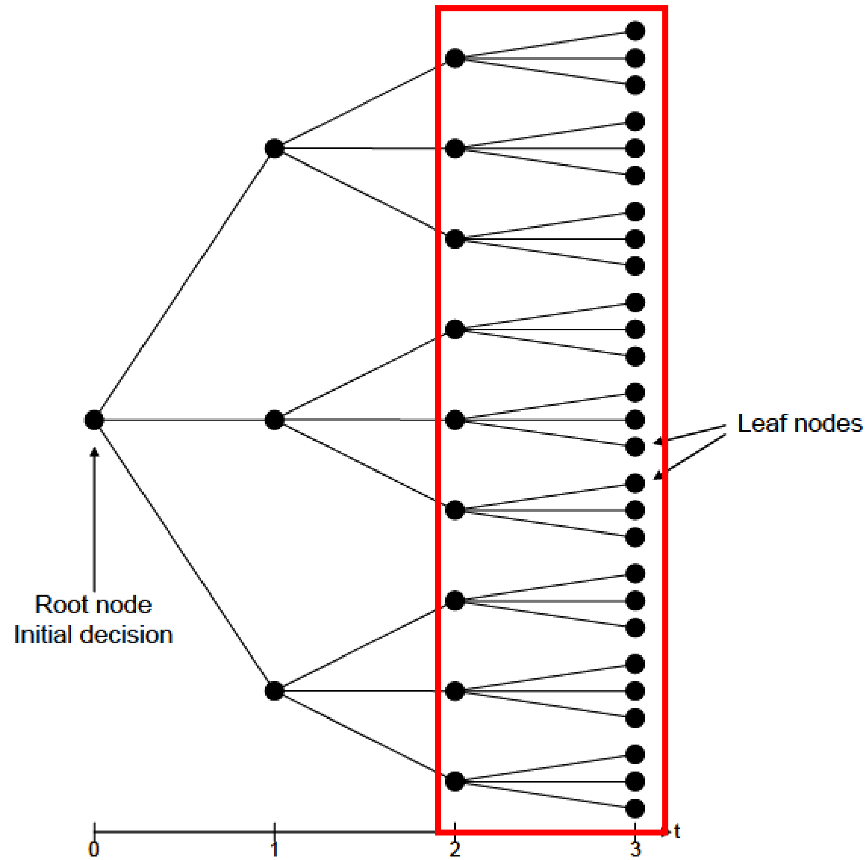
- **Step 2 in each iteration:** Solve 3 two-stage stochastic problems (separately)
- **Master problems**: Stage 2
- **Subproblems:** Stage 3



Some more thoughts

What to do in case of multi-stage (e.g., 4-stage) stochastic problem?

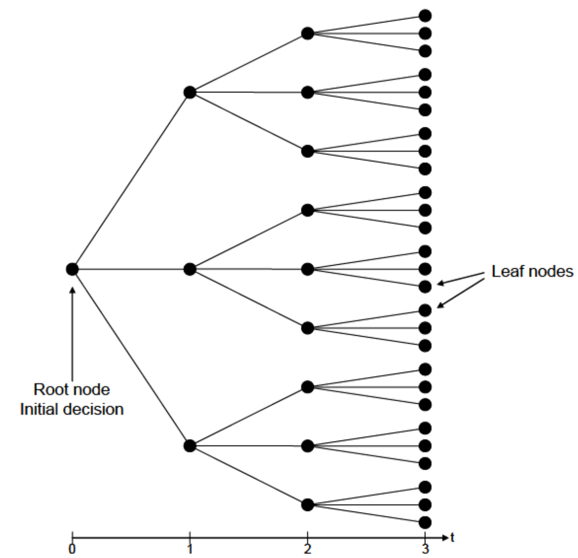
- **Step 3 in each iteration:** Solve 9 two-stage stochastic problems (separately)
- **Master problems**: Stage 3
- **Subproblems:** Stage 4



Some more thoughts

What to do in case of multi-stage (e.g., 4-stage) stochastic problem?

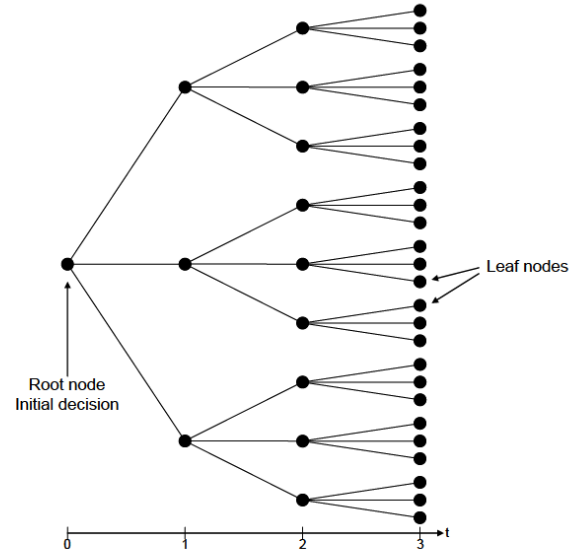
Each multi-stage stochastic problem is a collection of **nested** two-stage stochastic problems



Some more thoughts

What to do in case of multi-stage (e.g., 4-stage) stochastic problem?

Each multi-stage stochastic problem is a collection of **nested** two-stage stochastic problems



Techniques to be used:

- Nested Benders' decomposition
- Stochastic dual dynamic programming (SDDP)

Reference: J. Murphy, "Benders, Nested Benders and Stochastic Programming: An Intuitive Introduction", *Cambridge University Engineering Department Technical Report*, 2013.

Some more thoughts

What to do in a case in which subproblems include **binary (0/1)** variables?

Some more thoughts

What to do in a case in which subproblems include **binary (0/1)** variables?

The trouble?

Sensitivities (dual variables) cannot be obtained in a discrete feasible region! But, we need those sensitivities to generate cuts in the master problem!

Some more thoughts

What to do in a case in which subproblems include **binary (0/1)** variables?

Solution:

- Generate cuts in the master problem based on the values obtained for **"primal" variables, and not "dual" variables** of the subproblems!

Some more thoughts

What to do in a case in which subproblems include **binary (0/1)** variables?

Solution technique:

- Primal Benders' decomposition (cutting-plane method)

  This technique has recently been used in power systems applications for two-stage robust optimization problems

Reference: M. Zugno and A. J. Conejo, "A robust optimization approach to energy and reserve dispatch in electricity markets," *Eur. J. of Oper. Res.*, vol. 247, pp. 659-671, 2015.

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Thanks for your attention!

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