Optimal reserve offer deliverability in co-optimized electricity markets: A two-stage robust optimization approach

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Contents

• Introduction

• Robust model

• Solution approach

• Numerical results

• Conclusions
Introduction

• Power system operation is exposed to uncertainty sources
  □ Fluctuations in nodal net injections (demand, renewable-based generation)
  □ Loss of system components

• Capability to withstand uncertainty realizations implemented via reserves
Reserves

- Different types of reserves according to practical operating manuals

- One general classification:
  - Synchronized or spinning reserves \(\Rightarrow\) Provided by units that are scheduled on
  - Non-synchronized or non-spinning reserves \(\Rightarrow\) Provided even by units that are scheduled off
Trading reserves

- Reserves are considered as tradable commodities in a similar way to energy

- Market participants are allowed to submit reserve offers

- Typical reserve offer ⇒ Pair quantity-price

- Market operator is responsible for scheduling energy and reserves
Trading reserves

Producer 1
Energy & reserve offer

Producer i
Energy & reserve offer

Producer n
Energy & reserve offer

Market operator (ISO)
Market-clearing procedure

Awarded offers and bids

Energy bid & reserve offer
Consumer 1

Energy bid & reserve offer
Consumer j

Energy bid & reserve offer
Consumer n

Market-clearing prices
Trading reserves

- Current market implementations:
  - Sequential markets for energy and reserves (Europe) ⇒ Simple albeit suboptimal
  - Co-optimized electricity markets (Greece, USA) ⇒ Energy and reserves are jointly cleared
Goal of the co-optimization

• Determination of the awarded levels of energy and reserve offers that:
  
  - Maximize declared social welfare
  
  - Comply with operational limits
  
  - Guarantee power balance under the normal state and under any uncertainty realization
Problem definition

- Two different categories of system states:
  - Normal state $\Rightarrow$ Forecast uncertainty realizations
  - States under uncertainty
- Straightforward solution $\Rightarrow$ Replication of operational constraints (contingency-constrained model)
Contingency-constrained model

Minimize \( c(x, r) \)

subject to:

\[ g(x, r) \leq 0 \]

\[ g_k(x, r, x_k) \leq 0 \quad \forall k \in \mathcal{C} \]

- Large-scale mixed-integer program

Social welfare (energy and reserves)

Pre-contingency constraints

Post-contingency constraints
Contingency-constrained model

- Operational constraints (pre- and post-contingency):
  - Generation limits
  - Reserve limits
  - Network-related constraints $\Rightarrow$ dc load flow

- Mixed-integer linear programming
Contingency-constrained model

- Uncertainty realizations (nodal net injections and component outages) are characterized as contingencies indexed by $k$ within the contingency set $\mathcal{C}$

  - $D^k_{bt} \Rightarrow$ Realizations of nodal net injections

  - $A^k_{it}, A^k_{lt} \Rightarrow$ 0/1 parameters modeling the unavailability/availability of generators and lines
Contingency-constrained model

- Contingency set $\mathcal{C}$ determined by:
  - Plausible realizations of nodal net injections
    $$D_{bt}^k \in \mathcal{D}_{bt}$$
  - Security criterion ($n - 1, n - 2, n - K, n - K^G - K^L$)
    $$f \left( \{A_{it}^k\}_{i \in I}, \{A_{lt}^k\}_{l \in L} \right) \leq 0$$
Contingency-constrained model
Practical issues

- Problem dimension depends on the number of states
- Contingency set $\mathcal{C}$ is infinitely large
- Intractable model in practice
Practical solution

- Pre-specification of system reserve requirements based on engineering judgment

- Operation under uncertainty realization typically disregarded $\Rightarrow$ Use of forecast values
Practical solution for energy and reserve co-optimization

\[ \text{Minimize } c(x, r) \]

subject to:

\[ g(x, r) \leq 0 \]

\[ g^{req}(r) \leq 0 \]

• Drawback ⇒ Feasible deployment of reserve offers is not guaranteed even for sufficient reserve requirements!!
Contents

• Introduction

• Robust model

• Solution approach

• Numerical results

• Conclusions
Motivation for robust optimization

- Exact albeit intractable contingency-constrained model is essentially a worst-case problem

- Equivalent to a penalized contingency-constrained model
Motivation for robust optimization
Equivalent penalized model

- Nodal power imbalance is allowed $\Rightarrow$ Slack variables
- Penalty term in the objective function associated with the worst-case system power imbalance
- Equivalence guaranteed by a sufficiently large penalty coefficient
Motivation for robust optimization
Equivalent penalized model

Minimize \( c(x, r) + C^I \max_{k \in \mathcal{C}} (e^T \Delta P_k) \)

subject to:
\( g(x, r) \leq 0 \)
\( g_k(x, r, x_k, \Delta P_k) \leq 0 \quad \forall k \in \mathcal{C} \)

- Also a large-scale mixed-integer program
Two-stage adaptive robust optimization

- Worst-case optimization setting ⇒ Suitable for ensuring feasibility under all uncertainty realizations
- No accurate probabilistic information is required ⇒ Suitable for renewable-based generation
- Problem size not dependent on the level of accuracy for uncertainty ⇒ Convenient for tractability purposes
Two-stage adaptive robust optimization

- First stage:
  - Decisions that immunize against uncertainty realizations

- Second stage:
  - Worst-case uncertainty realizations and corresponding corrective actions
Uncertainty characterization

• Uncertainty realizations are represented by uncertainty-related decision variables

  - $d_{bt}$ ⇒ Nodal net injections
  - $a_{it}^G, a_{lt}^L$ ⇒ Loss of generators and lines

• Non-dependent on index $k$!!
Uncertainty characterization

- Feasibility space of uncertainty-related decision variables $\Rightarrow$ Pre-specified uncertainty set $\mathcal{U}$

$$a_{it}^G, a_{lt}^L, d_{bt} \in \mathcal{U}$$
Uncertainty set

• Upper and lower bounds for uncertainty-related decision variables ⇒ Based on historical data or practical aspects

• Uncertainty budget ⇒ Limit on the conservativeness of the solution based on engineering judgement
Uncertainty set: Nodal net injections

- Cardinality model:

\[ d^f_{bt} - \Delta^d_{bt} \leq d_{bt} \leq d^f_{bt} + \Delta^u_{bt} \]

\[ \sum_{b \in B^u} \left[ \frac{\max\{0, d_{bt} - d^f_{bt}\}}{\Delta^u_{bt}} + \frac{\max\{0, d^f_{bt} - d_{bt}\}}{\Delta^d_{bt}} \right] = K \]

- Tradeoff between accuracy and complexity
Uncertainty set: Nodal net injections

- Polyhedral uncertainty sets:
  - Worst-case uncertainty realization $\Rightarrow$ Extreme of the polytope
  - Equivalent discrete model based on binary variables
Uncertainty set: Nodal net injections

- Binary-variable-based cardinality model:

\[ d_{bt} = d^f_{bt} + \Delta^{up}_{bt} a^{up}_{bt} - \Delta^{dn}_{bt} a^{dn}_{bt} \]

\[ \sum_{b \in B^u} (a^{up}_{bt} + a^{dn}_{bt}) = K \]

\[ a^{up}_{bt}, a^{dn}_{bt} \in \{0,1\} \]
Uncertainty set: Component outages

- Straightforward use of 0/1 variables $a_{it}^G$ and $a_{it}^L$
- Uncertainty budget $\Rightarrow$ Security criterion
- Binary-variable-based cardinality model:

$$a_{it}^G, a_{it}^L \in \{0,1\}$$

$$f \left( \{a_{it}^G\}_{i \in I}, \{a_{lt}^L\}_{l \in L} \right) \leq 0$$
Robust counterpart

- Contingency constraints are replaced with an optimization problem to characterize the worst case

- Worst case $\Rightarrow$ Maximum damage (system power imbalance) associated with ALL contingencies implicitly modeled by $a_{bt}^{up}$, $a_{bt}^{dn}$, $G_u$, $G_l$, $L_u$, $L_l$,

- Robust counterpart $\Rightarrow$ Trilevel program
Trilevel robust counterpart

Maximize social welfare
Determine: On/off decisions
Power and reserves schedule

Pre-contingency schedule

Maximize the system power imbalance given the scheduled power and reserves
Determine: Nodal net injections, unavailable components

Worst-case contingency

Minimize the system power imbalance given the scheduled power and reserves and the worst-case contingency
Determine: Corrective actions

Operator’s reaction

First stage

Second stage
Trilevel robust counterpart

\[ \text{Minimize}_{x,r} \ c(x, r) + C^I \Phi^{wc}(x, r) \]

subject to:
\[ g(x, r) \leq 0 \]
\[ \Phi^{wc}(x, r) = \max_a \ \delta(x, r, a) \]

subject to:
\[ f^{dis}(a) \leq 0 \]
\[ \delta(x, r, a) = \min_{x^u, \Delta P^u} \ (e^T \Delta P^u) \]

subject to:
\[ g^u(x, r, a, x^u, \Delta P^u) \leq 0 \]
Trilevel robust counterpart

- Two lowermost optimization problems replace post-contingency constraints $\Rightarrow$ Mixed-integer trilevel program

- Penalty term in the upper-level objective function is optimized in the two lowermost problems

- Two lowermost optimization problems $\Rightarrow$ Max-min optimization with a linear lower-level problem
Contents

- Introduction
- Robust model
  - Solution approach
- Numerical results
- Conclusions
Suitable solution approaches

- Decomposition-based methods involving the iterative solution of a master problem and a max-min subproblem:
  - Benders decomposition
  - Column-and-constraint generation algorithm

- Optimal solution to the master problem and the max-min subproblem $\Rightarrow$ Finite convergence to global optimality
Column-and-constraint generation
Master problem

- Relaxation of the trilevel counterpart
- Iteratively tighter by the addition of operating constraints built with information from the subproblem
Column-and-constraint generation
Master problem

\[ \text{Minimize}_{\alpha, x, r, x^u, m, \Delta P^u, m} \ c(x, r) + C^T \alpha \]

subject to:

\[ g(x, r) \leq 0 \]

\[ \alpha \geq e^T \Delta P^u, m; \ \forall m \in \mathcal{M} \]

\[ g^u(x, r, a^{(m)}, x^u, m, \Delta P^u, m) \leq 0; \ \forall m \in \mathcal{M} \]
Column-and-constraint generation
Max-min subproblem

- Two lowermost levels for given $x$ and $r$ from the master problem

$$\text{Maximize}_a \ \delta(x, r, a)$$
subject to:

$$f^{\text{dis}}(a) \leq 0$$
$$\delta(x, r, a) = \min_{x^u, \Delta P^u} (e^T \Delta P^u)$$
subject to:

$$g^u(x, r, a, x^u, \Delta P^u) \leq 0$$
Column-and-constraint generation
Max-min subproblem

- Transformation to a single-level mixed-integer linear equivalent
  - Dual of the lower-level problem renders the original max-min a max-max $\Rightarrow$ Maximization problem (strong duality theorem)
  - Integer algebra results for the resulting bilinear terms
Dual of the lower-level problem

\[ \delta(x, r, a) = \max_{\pi} h^{u,dual}(\pi, x, r, a) \]

subject to:

\[ g^{u,dual}(\pi, a) \leq 0 \]

- At the optimum (strong duality theorem):

\[ (e^T \Delta P u) = h^{u,dual}(\pi, x, r, a) \]
Single-level equivalent for the subproblem

Maximize \(a, \pi \) \(h^{u, \text{dual}}(\pi, x, r, a)\)

subject to:
\[f^{\text{dis}}(a) \leq 0\]
\[g^{u, \text{dual}}(\pi, a) \leq 0\]

• Bilinear terms in \(h^{u, \text{dual}}(\pi, x, r, a)\) ⇒ Well-known linearization scheme
Column-and-constraint generation

MASTER PROBLEM

Power and reserves schedule
Lower bound for social welfare

SUBPROBLEM

Worst-case contingency
Upper bound for social welfare

Add a set of lower-level constraints
Contents

• Introduction
• Robust model
• Solution approach
• Numerical results
• Conclusions
Case studies

- Illustrative three-bus, three-line, single-period example
- IEEE 118-bus system, 24 hourly periods
- CPLEX 12.6 under GAMS 24.2
- 4 Intel Xeon E7-4820 processors, 2.00 GHz, 768 GB of RAM
Illustrative single-period example

Forecast generation = 10 MW

$P_{max} = 100\text{ MW}$

Bus 1

Unit 1

20 MW

$P_{max} = 100\text{ MW}$

Unit 2

Bus 2

$P_{max} = 30\text{ MW}$

Bus 3

Unit 3

130 MW

Forecast generation = 65 MW
## Illustrative single-period example

<table>
<thead>
<tr>
<th>Unit</th>
<th>$\bar{P}, \bar{R}$ (MW)</th>
<th>$\bar{P}$ (MW)</th>
<th>$C^v$ ($/\text{MWh}$)</th>
<th>$C^f$ ($)</th>
<th>$C^{up}$ ($)</th>
<th>$C^{dn}$ ($)</th>
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<tbody>
<tr>
<td>1</td>
<td>50</td>
<td>2.4</td>
<td>50.0</td>
<td>10.0</td>
<td>1.0</td>
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<tr>
<td>2</td>
<td>50</td>
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<td>0.5</td>
<td>0.3</td>
<td>0.02</td>
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<tr>
<td>3</td>
<td>50</td>
<td>2.4</td>
<td>19.9</td>
<td>5.0</td>
<td>0.7</td>
<td>0.10</td>
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</table>

- All line reactances equal to 0.63 p.u.
- $\pm 40\%$ wind power fluctuation ($\pm 30$ MW), no component outages
### Illustrative single-period example

#### Results

<table>
<thead>
<tr>
<th>Unit</th>
<th>Under no uncertainty</th>
<th>Under uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( p_i ) (MW)</td>
<td>( p_i ) (MW)</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>2.4</td>
</tr>
<tr>
<td>2</td>
<td>50</td>
<td>48.8</td>
</tr>
<tr>
<td>3</td>
<td>25</td>
<td>23.8</td>
</tr>
</tbody>
</table>

- Convergence in 3 iterations requiring less than 1 second

| Cost ($) | 653.0 | 778.1 | 777.4 |
Illustrative single-period example
Worst-case operation

Robust solution

Conventional solution
IEEE 118-bus system

- 118 buses, 186 lines, 54 dispatchable units, 10 wind farms (25.8% wind penetration), 24 periods

- ±20% wind power fluctuation

- 66 contingencies (all dispatchable generators + 12 critical lines)

- 0.01-MW stopping criterion
IEEE 118-bus system
Results

- Convergence in 8 iterations, 2.2 min/iteration
IEEE 118-bus system
Infeasibility of reserve requirements

<table>
<thead>
<tr>
<th>$K$</th>
<th>Worst power Imbalance (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1980.3</td>
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<tr>
<td>2</td>
<td>2675.5</td>
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<tr>
<td>3</td>
<td>3049.5</td>
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<td>4</td>
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<td>5</td>
<td>3908.8</td>
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<td>6</td>
<td>4328.4</td>
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<tr>
<td>7</td>
<td>4718.0</td>
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<tr>
<td>8</td>
<td>5107.6</td>
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<tr>
<td>9</td>
<td>5492.2</td>
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<tr>
<td>10</td>
<td>5863.4</td>
</tr>
</tbody>
</table>

- Feasibility $\implies$ 0.1%-6.8% cost increase
Contents

• Introduction

• Robust model

• Solution approach

• Numerical results

• Conclusions
Conclusions

• Robust optimization is suitable for co-optimized electricity markets under uncertainty

• Reserve deliverability is achieved with moderate reduction of social welfare

• Optimality is attained in a few iterations
Further research

- Incorporation of correlated uncertainty sources ⇒ Alternative uncertainty sets

- Consideration of more sophisticated operational models (ac load flow, line switching)

- Analysis of alternative solution approaches to avoid the dual-based transformation
End of the presentation

Thanks for your attention!

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